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MIA-005 F2F

M.Sc. ACTUARIAL SCIENCE

Term-End Examination

June, 2011

MIA-005 F2F : STOCHASTIC MODELLING AND SURVIVAL MODELS

Time : 3 hours *Maximum Marks : 100*

Note : *Students are allowed to use their* ***actuarial tables*** *and calculators (scientific).*

SECTION - A 5x8=40

(Answer ***any five*** *questions)*

1. (a) List the benefits and limitations of modelling in actuarial work. **4**

(b) Define each of the following examples of a stochastic process : **4**

(i) white noise

(ii) poisson process

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2. For a particular data entry operator, any period j is either error free ($y_j = 0$) or give rise to one error ($y_j = 1$). The probability of having no error in the next period is estimated using the operator's past record as follows all values of y_j are either 0 or 1) :

$$p\ [y_{n+1} = 0 / y_1 = y_1,\ y_2 = y_2,\ ..\ y_n = y_n]$$

$$= pe^{-\lambda (y_1 + y_2 + .. + y_n)}$$

Where $0<p<1$ and $\lambda \geq 0$. The cumulative number of errors committed by the operator over the time period 1 to n is given by

$$X_n = \sum_{j=1}^{n} y_j .$$

(a) Verify that the Markov property holds for the sequence $\{X_n : n \geq 1\}$. **2**

(b) Explain why the sequence $\{Y_n : n \geq 1\}$ does not form a Markov Chain. **1**

(c) Write down the transition matrix for the Markov Chain x. **2**

(d) State with reasons the following : **3**

(i) Whether the Markov Chain X is time homogeneous

(ii) Whether the Markov Chain X is irreducible

(iii) Whether the Markov Chain X admits a stationary probability distribution.

3. For a discrete time stochastic process X_n, define the terms :

(a) Stationary **2**

(b) Weakly stationary **2**

(c) Increment **2**

(d) Markov property **2**

4. (a) Assuming uniform distribution of deaths between age x and $x+1$, show that :

(i) $M_{x+t} = \dfrac{q_x}{1 - tq_x}$ **2**

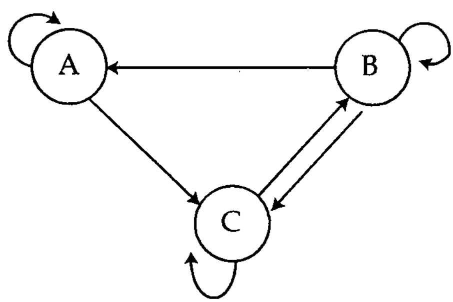
(ii) ${}_{t-s}q_{x+s} = \dfrac{(t-s)\, q_x}{1 - s\, q_x}$ for $s + t \leq 1$ **2**

(b) Given $p_{56} = 0.990581$, $p_{57} = 0.989503$ and $p_{58} = 0.988314$. Find $2P_{56.75}$ assuming

(i) A uniform distribution of deaths between integral ages. **2**

(ii) A constant force of mortality between integral ages **2**

5. A three state process with state space {A, B, C} is believed to follow a Markov chain with the following possible transitions : **8**



An instrument was used to monitor this process, but it was set up incorrectly and only recorded the state occupied after every two time periods. From these observations the following two-step transition probabilities have been estimated :

$P^2_{AA} = 0.5625$, $P^2_{AB} = 0.125$

$P^2_{BA} = 0.475$, $P^2_{CC} = 0.4$

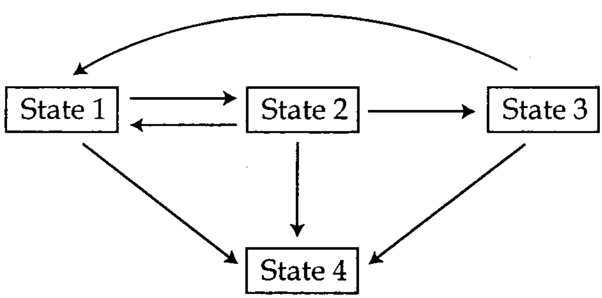
Calculate the one step transition matrix consistent with these estimates.

6. A mortality investigation was held between 1 January 2007 and 1 January 2009. The following information was collected. The figures in the table below are the numbers of lives on each census date with the specified age labels 8

Age last birthday	Date		
	1.1.07	1.1.08	1.1.09
48	3,486	3,384	3,420
49	3,450	3,507	3,435
50	3,510	3,595	3,540

During the investigation there were 42 deaths at age 49 nearest birthday. Estimate μ_{49} stating any assumption that you make.

7. Consider the following multi-state model in which S (t), the state occupied at time t by a life initially aged x, is assumed to follow a continuous-time Markov process.



Let μ_{x+t}^{ij} denote the force of transition at age $x+t$ $(t \geq 0)$ from state i to state j, and let ${}_tp_x^{ij} = p\ (s(t) = j \mid s(0) = i)$.

(a) Derive the forward differential equation : **5**

$$\frac{\partial}{\partial t}\, {}_tp_x^{21} = {}_tp_x^{22}\, \mu_{x+t}^{21} + {}_tp_x^{23}\, \mu_{x+t}^{31}$$

$$- {}_tp_x^{21}\left(\mu_{x+t}^{12} + \mu_{x+t}^{14}\right)$$

stating all the assumption that you make.

(b) Write down forward differential equations for ${}_tp_x^{23}$ and ${}_tp_x^{32}$. **3**

SECTION - B

(Answer any four questions)

8. An investigation into the mortality of patients following a specific type of major operation was undertaken. A sample of 10 patients was followed from the date of the operation until either they died, or they left the hospital where the operation was carried out, or a period of 30 days had elapsed (whichever of these events occurred first). The data on the 10 patients are given in the table below.

Patient number	Duration of observation (days)	Reason for observation ceasing
1	2	Died
2	6	Died
3	12	Died
4	20	Left hospital
5	24	Left hospital
6	27	Died
7	30	Study ended
8	30	Study ended
9	30	Study ended
10	30	Study ended

(a) State whether the following types of censoring are present in this investigation. In each case give a reason for your answer. **3**

(i) Type I

(ii) Type II

(iii) Random

(b) State, with a reason, whether the censoring in this investigation is likely to be informative 2

(c) Calculate the value of the Kaplan-Meier estimate of the survival function at duration 28 days. 6

(d) Write down the Kaplan-Meier estimate of the hazard of death at duration 8 days. 1

(e) Sketch the Kaplan-Meier estimate of the survival function. 3

9. A motor insurer operates a no claims discount system with the following levels of discount {0%, 25%, 50%, 60%}.

The rules governing a policy holder's discount level, based upon the number of claims made in the previous year, are as follows :

- Following a year with no claims, the policy holder moves up one discount level or remains at 60% level.
- Following a year with one claim, the policy holder moves down one discount level, or remains at 0% level.
- Following a year with two or more claims, the policy holder moves down two discount levels (subject to a limit of the 0% discount level)

The number of claims made by a policy holder in a year is assumed to follow a poisson distribution with mean 0.30.

(a) Determine the transition matrix for the no claims discount system 3

(b) Calculate the stationary distribution of the system, π. 5

(c) Calculate the expected average long term level of discount. 1

The following data shows the number of insurer's 130,200 policy holders in the portfolio classified by the number of claims each policy holder made in the last year. This information was used to estimate the mean of 0.30.

No claims	96,632
One claim	28,648
Two claims	4,400
Three claims	4,76
Four claims	36
Five claims	8

(d) Test the goodness of fit of these data to a poisson distribution with mean 0.30. **5**

(e) Comment on implications of your conclusion in (d) for the average level of discount applied. **1**

10. An education authority provides children with musical instrument tuition. The authority is concerned about the number of children giving up playing their instrument and is testing a new tuition method with a proportion of the children which it hopes will improve persistency rates. Data have been collected and a cox proportional hazard model has been fitted for the hazard of giving up playing the instrument. Symmetric 95% confidence intervals (based upon standard errors) for the regression parameters are shown below.

Covariate	Confidence Interval
Instrument	
Piano	0
Violin	[− 0.05, 0.19]
Trumpet	[0.07, 0.21]
Tuition method	
Traditional	0
New	[− 0.15, 0.05]
Sex	
Male	[− 0.08, 0.12]
Female	0

(a) Write down a general expression for the cox proportional hazards model defining all terms that you use. **4**

(b) State the regression parameters for the fitted model. **3**

(c) Describe the class of children to which the baseline hazard applies. **1**

(d) Discuss the suggestion that the new tuition method has improved the chances of children continuing to play their instrument. **3**

(e) Calculate, using the results from the model, the probability that a boy will still be playing the piano after 4 years, if provided with the new tuition method given that the probability that a girl will still be playing the trumpet after 4 years following the traditional method is 0.7. **4**

11. (a) The probability that exactly x decrements will occur in a population consisting initially of n individuals subject to a single decrement with rate q per annum is : **3**

$$\binom{n}{x} q^{x} (1-q)^{n-x}$$

(b) The maximum likelihood estimator of the parameter q for the binomial model equals the number of decrements divided by the initial population. **4**

(c) (i) Describe the effect of duplicate policies on the number of claims and its variance. **4**

(ii) Calculate the variance ratio for the following data : **4**

No of policies held by an individual	Number of individuals
1	8000
2	1000
3	500
4	300
5	200

Calculate the variance of the number of policies resulting into claims by death assuming all policyholders are ages 55 years and Mortality table is AM-92 (Vlt.)

12. (a) (i) Explain what is meant by a Markov jump process. **1**

(ii) Explain the condition needed for such a process to be time-homogeneous **1**

(iii) Outline the principal difficulties in fitting a Markov Jump Process Model with time-inhomogeneous rates. **2**

(b) A particular machine is in constant use. Regardless of how long it has been since the last repair, it tends to break down once a day and on average it takes the repairman 6 hours to fix. You are modelling the machine's status as a time-homogeneous Markov jump process $\{x\ (t) : t \geq 0\}$ with two states : "being repaired" denoted by 0, and "working" denoted by 1. Let $P_{i,j}\ (t)$ denote the probability that the process is in state j at time t given that it was in state i at time 0 and suppose that t is measured in days.

(i) State the two main assumptions that you make in applying the model and discuss briefly how you could test that each of them hold. **4**

(ii) Draw the transition graph for the process, showing the numerical values of the transition rates. **2**

(iii) State Kolmogorov's backward and forward differential equations for the probability $P_{0,0}(t)$. **2**

(iv) Solve the forward differential equation in (c) to show that **3**

$$P_{0,0}(t) = \frac{1}{5} + \frac{4}{5}e^{-5t}$$

13. (a) Explain why graduated rates, rather than crude estimates of mortality rates are used in the construction of standard mortality tables. **3**

(b) A graduation of the mortality experience of the male population of a region of the united kingdom has been carried out using a graphical method. The following is an extract from the results. **4**

Age	Actual number of deaths	Graduated mortality rate	Initial exposed to risk	
x	θ_x	$°q_x$	E_x	$E_x q°_x$
14	3	0.00038	12,800	4.86
15	8	0.00043	15,300	6.58
16	5	0.00048	12,500	6.00
17	14	0.00053	15,000	7.95
18	17	0.00059	16,500	9.74
19	9	0.00066	10,100	6.67
20	15	0.00074	12,800	9.47
21	10	0.00083	13,700	11.37
22	10	0.00093	11,900	11.07
Total	91			73.71

Use the chi-squared test to test the adherence of the graduated rates to the data. State clearly the null hypothesis you are testing and comment on the result.

(c) Perform two other tests which detect different aspects of the adherence of the graduation to the data. For each test state clearly the features of the graduation which the test is able to defect, and comment on your result. 8