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M.Sc. ACTUARIAL SCIENCE**Term-End Examination****June, 2011****MIA-001 F2F : FINANCIAL MATHEMATICS***Time : 3 hours**Maximum Marks : 100*

Note : In addition to this paper you should have available actuarial tables and an electronic calculator.

SECTION - A**5x8=40**(Answer *any five* questions)

1. Given $d^{(2)} = 0.075$, find
- (a) $G_{\overline{10}|}^{(3)}$ 2
- (b) $\ddot{S}_{\overline{9}|}$ 2
- (c) $(Ia)_{\overline{15}|}$ 2
- (d) $(Da)_{\overline{12}|}$ 2
2. (a) Explain when you would use real and money rate of interest. 2
- (b) Suppose that the force of interest per annum at time t years is $\delta(t) = r + S e^{-rt}$ 6
Show that the present value of an annuity payable continuously for n years at a constant rate of Rs. 1000 per annum is :

$$\frac{1000}{S} \left\{ 1 - \exp \left[\frac{S}{r} (e^{-rn} - 1) \right] \right\}$$

3. (a) Prove that 3

$$\ddot{S}_{\overline{n}|}^{(P)} = \frac{(1+i)^n - 1}{d^{(P)}}$$

- (b) Let i_t denote the rate of interest earned in the year $t-1$ to t . Each year the value of i_t is 8% with probability 0.625, 4% with probability 0.25 and 2% with probability 0.125. In any year, it is independent of the rates of interest earned in previous years. Let S_3 denote the accumulated value of 1 unit for 3 years. Calculate the mean and standard deviation of S_3 . 5
4. A man invested Rs. 20,000 at time 1, 3 and 4. In return he received Rs. 8000 *pa* monthly in arrears for 15 years starting at time 4. Calculate the discounted payback period and the accumulated profit after he has received his last monthly payment, assuming an effective annual rate of interest of 6%. 8
5. The liabilities of a fund consist of two lump sum payments due at known times in the future. The second lump sum is due for payments 5 year after the first and is twice the amount of the first.
- (a) If the total present value and the duration of the liabilities (both calculated using a market interest rate of 6%) are Rs. 75,000 and 8 years respectively, determine the precise timing and amounts of the payments. 5
- (b) If the assets of the fund consist of a single zero coupon bond that will mature 8 years from now with a redemption payments of Rs. 1,19,540, what you say about this portfolio on the basis of Redington's theory of immunisation ? 3

6. Short-term, one-year annual effective interest rates are currently 10% ; they are expected to be 9% in one year's time, 8% in two years time, 7% in three years time and to remain at that level there after indefinitely.
- (a) If bond yields over all terms to maturity are assumed only to reflect expectations of future short-term interest rates, calculate the gross redemption yield from 1-year, 3-year, 5-year and 10-year zero coupon bonds. 4
- (b) Draw a rough plot of the yield curve for zero coupon bonds using the data from part (i). (Graph paper is not required) 2
- (c) Explain why the gross redemption yield curve for coupon paying bonds will slope down with a less steep gradient than the zero coupon bond yield curve. 2
7. (a) (i) Define the characteristic of government index linked bond. 2
- (ii) Explain why most index-linked securities issued carry some inflation risk in practice. 2
- (b) Dividend payable on a certain share are assumed to increase at a compound rate of 3% per half-year. A dividend of Rs. 2.50 per share has just been paid. Dividend are paid half-yearly. Find the value of the share to the nearest rupee, assuming an effective rate of interest of 8% per annum. 4

SECTION - B

4x15=60

(Answer *any four* questions)

8. (a) The force of interest takes the following values
- $$\delta(t) = 0.04 \quad 0 < t \leq 10$$
- $$\delta(t) = 0.001(t-10)^2 + 0.04 \quad 10 < t$$
- (i) Calculate the accumulation of Rs. 150 from time $t=0$ to time $t=20$. 4
- (ii) A continuous payment stream of 10 per annum is received from time $t=5$ to time $t=10$. Calculate the present value of that payment stream at time $t=0$. 4
- (iii) A continuous payment stream is paid at rate $e^{-0.03t}$ from time $t=0$ to time $t=10$. Calculate the present value of that payment stream at time $t=0$. 3
- (b) A customer borrows Rs. 4000 under a consumer credit loan. Repayments are calculated to give an APR of 15.4%. Instalments are paid monthly in arrears for 5 years. Calculate the flat rate of interest. 4
9. A loan is repayable by an increasing annuity payable annually in arrears for 15 years. The repayment at the end of first year is Rs. 2,00,000 and subsequent repayments increase by Rs. 20,000 every year. The repayments were calculated using a rate of interest of 12% per annum effective.
- (a) Calculate the amount of loan originally granted. 4

- (b) Construct the schedule of repayment for years nine and ten [i.e immediately after the 8th repayment instalment], showing the outstanding loan at the beginning of the year, the repayment instalment, interest portion and capital repayment portion in each year. 5
- (c) Immediately after the tenth instalment of repayment, the rate of interest on the outstanding loan is reduce to 10% per annum effective. Calculate the amount of the eleventh repayment instalment if the subsequent instalments continue to increase at Rs. 20,000 each year and the loan is repaid by the original date is 15 years from the commencement. 6
10. (a) On 1st January 1996, an investor placed part of his assets with two fund managers. Manager A was given Rs. 1,20,000 and manager B was given Rs. 1,00,000. Rs. 10,000 invested with each manager on 1 January 1997 and a further Rs. 10,000 invested with each manager on 1st January 1998. The values of the respective funds on 31 December 1996 were Rs. 1,30,000 for manager A and Rs. 1,40,000 for manager B and on 31 December 1997, the values were Rs. 1,35,000 for manager A and Rs. 1,45,000 for manager B. On 31st December 1998, the value of manager A's fund was Rs. 1,80,000 and of manager B's fund was Rs. 1,50,000.

- (i) Calculate the time weighted rate of return earned by manager A and manager B over the period 1 January 1996 to 31 December 1998. 4
- (ii) Calculate the money weighted rate of return earned by manager A over the period 1st January 1996 to 31 December 1998. 3
- (iii) Without calculating the money weighted rate of return earned by manager B, state, with reasons, whether the money weighted rate of return earned by manager B over the period 1st January 1996 to 31st December 1998 is higher than, equal to or lower than that earned by manager A. 3
- (b) What price should an investor who requires a yield of 10% per annum and pay 33% tax on income and capital gains pay on 15 July 2004 for Rs. 100 nominal of a six-year stock with half yearly coupon payments of 13% *pa* ? 5
11. (a) In the context of a derivatives market define the terms. 3
- (i) call option
- (ii) put option
- (iii) long party
- (b) (i) Define arbitrage and describe when an arbitrage opportunity may exist. 3
- (ii) Describe how the principle of 'no arbitrage' is used to find the price of complex financial instruments. 3

- (c) How will the price of a convertible gilt that is redeemable at par with an annual coupon of 3% be affected if future rate of interest over all term increase from 7% to 8%. If the term of the gilt is : 6

- (i) 5 years
(ii) 10 years

Comment on your results.

12. (a) The n -year spot rate of interest S_n is given by the formula

$$S_n = 0.05 - \frac{n}{500} \text{ for } n = 1, 2 \text{ and } 3$$

- (i) Calculate the implied one year forward rates applicable at time $t=1$ and 2. 3
- (ii) An investor purchase a 3-year bond that provides coupons of 6% *pa* payable annually in arrears and is redeemable at par. Show that the fair price for this bond is Rs. 104.36 per Rs. 100 nominal. 1
- (iii) Calculate the investor gross redemption yield. 3
- (iv) Calculate the par yield of the bond. 3
- (b) An investment provide income of Rs. 1000 payable at the end of each year for the next ten years. There is no capital repayment. If the interest rate is 7% per annum effective, show that the discount mean term of the investment is 4.946 years. 5

13. (a) Outline the similarities and differences between deterministic and stochastic interest rate model. 3
- (b) In any year, the yield on fund invested with a given insurance company has mean j and standard deviation S , and is independent of the yield in all previous years. Let i_t be the rate of interest earned in the t^{th} year. 12
- Each year the value of $(1 + i_t)$ is lognormally distributed. The rate of interest has a mean value of $J=0.06$ and standard deviation of $S=0.1$ in all years.
- (i) Find the parameter μ and σ^2 for the log normal distribution.
- (ii) Derive the distribution of S_{10} , where S_{10} denote the accumulated of one unit of money for 10 years.
- (iii) Find the single amount that should be invested to give an accumulation of at least Rs. 50,000 in 10 years time with probability 0.95.
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