00613

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination

June, 2011

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Do **any five** questions from questions **1** to **6**. Use of calculators **not** allowed.*

1. (a) Consider the [7, 4] binary code with the following generator matrix : **4**

$$\left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

(i) Write down the parity check matrix.

(ii) Find four information sets in the above code.

(iii) Find one set of 4 co-ordinates that do not form an information set.

(b) (i) Find the dimension and minimum weight of the Reed-Muller code R(2, 4). **6**

(ii) Find the generator matrix of the Reed - Muller code R(3, 4).

(iii) The parity check matrix of [15, 11) binary Hamming code is given below :

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Assume that the received vector is (0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1). Find the correct decoded message.

2. (a) (i) Show that the polynomial $f(x) = x^3 + x^2 + 1$ is irreducible in $F_2[x]$. 5

(ii) Let $\alpha = x + (f(x)) \in \dfrac{F_2[x]}{(f(x))}$.

Write every element of $\dfrac{F_2[x]}{(f(x))}$ as a power of α.

(iii) Write $\alpha^5 + \alpha^4 + \alpha^2 + 1$ as a power of α, where α is as in (i).

(b) Find all the codewords of cyclic code with generator matrix. 5

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Find the minimum weight of the code. How many errors can the code detect and how many can it correct ?

3. (a) Construct all possible BCH codes one F_B of length 8. **6**

(b) Let C be any $\left[n, \frac{(n-1)}{2}\right]$ cyclic code over F_q. Then show that C is self-orthogonal if and only if C is an even-like duadic code whose splitting is given by μ_{-1}. **4**

4. (a) Let A_i and A_i^1 be the number of code words of weight i in C and C^1, respectively. Let C be a binary code generated by **6**

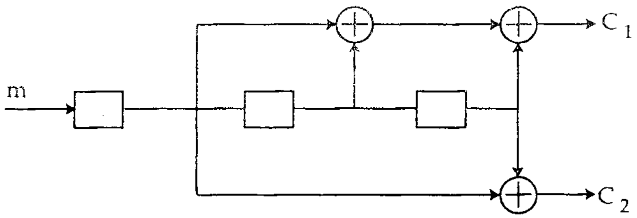
$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$ For $0 \le i \le 6$, find A_i and A_i^1.

(b) Show that the Z_4 - linear codes with generator matrices **4**

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

are monomially equivalent.

5. (a) Find the convolutional code for the message 1101. The convolutional encoder is given below : 4



(b) Explain the two way APP decoding algorithm of turbo codes. 6

6. (a) Let C be a [6, 3, 2] code with generator matrix 4

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(i) Find the generator matrix of the dual code C^1.

(ii) Find the generator matrix for a [4, 2, 3] code. Obtained by shortening the generator matrix for C.

(b) Find all the generator polynomials for a [7, 4] cyclic code. 4

(c) Compute the 3 - cyclotomic cosets modulo 8. 2
