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MMTE-001

M.Sc. (Mathematics with Applications in Computer Science) (MACS)

00323

Term-End Examination

June, 2011

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. **1** is **compulsory**. Answer **any four** from the remaining **six (2 - 7)**. Calculators and similar devices are **not** allowed.*

1. State, giving justifications or illustrations, whether each of the following statements is true or false. **5x2=10**

   (a) The complement of a connected simple graph need not be connected.

   (b) Degree sequence of any simple graph contains atleast one number appearing more than once.

   (c) Every edge cut is a disconnecting set.

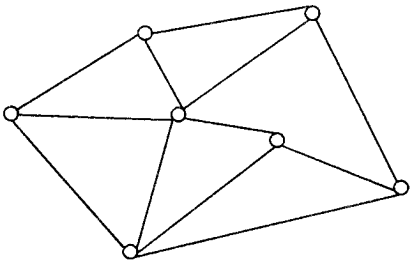
   (d) Any simple graph with atleast 4 vertices is 4-colorable.

   (e) If G is an Eulerian graph, then R(G)>1.

....

2. (a) Draw the diagram of a graph G with 8 vertices and 14 edges such that $\Delta(G) - \delta(G) \leq 1$ and check whether it is Eulerian or not. **4**

(b) Draw the dual graph of the following planar graph. **2**

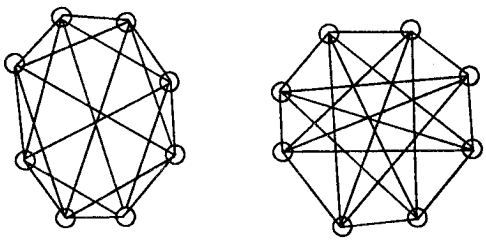


(c) Draw a cubic graph with $\kappa(G) = 1$. **4**

3. (a) How many edges will a planar graph with 8 vertices and 6 faces have ? Draw a planar graph with 8 vertices and 6 faces. **4**

(b) Construct a 3-regular simple graph having no one-factor. **3**

(c) Determine whether the graphs given below are isomorphic to each other. Justify your answer. **3**

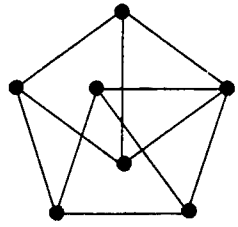


4. (a) Find the independence number of the Petersen Graph. Justify your answer. **3**

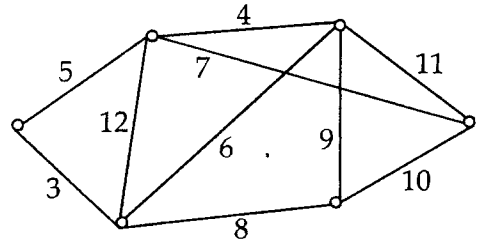
(b) Prove that, if u and v are the only vertices of odd in a graph G, then G contains a u, v-path. **3**

(c) (i) State Brook's theorem. **4**

(ii) Find the chromatic number of the graph below.

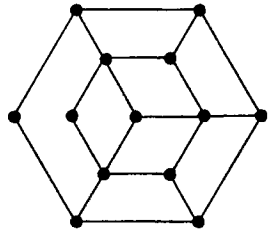


5. (a) Find the minimal spanning tree in the following graph : **4**



(b) (i) State any one necessary and any one sufficient condition to be satisfied for a graph to be Hamiltonian. 2

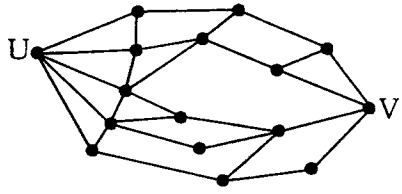
(ii) Check whether the following graph is Hamiltonian or not ? Give reasons for your answer. Also write down the Hamiltonian cycle if one exists. 4



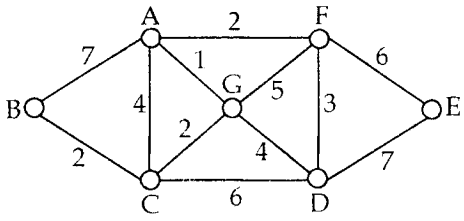
6. (a) Let G be a acyclic graph with n vertices and $(n-1)$ edges. Prove that G is connected. 3

(b) Prove or disprove : The sequence (5, 5, 5, 4, 2, 1, 1, 1) is a graphic sequence. 4

(c) Determine $k(u, v)$ for the following graph : 3



7. (a) Using Dijkstra's algorithm, find the shortest distance from vertex A to all the vertices in the following weighted graph : 4



(b) Prove that every tree is 2-colourable. 3

(c) Show that the petersen graph in 3-chromatic. 3