## M M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination<br>June, 2011

## MMT-009 : MATHEMATICAL MODELLING

Time : $11 / 2$ hours
Maximum Marks : 25
Note: Do any five questions. Use of calculator is not allowed.

1. (a) Compare the risk of two securities 1 and $2 \quad 3$ whose return distributions are given below :

| Possible rates of returns <br> for Security | Associated <br> Probability |  |
| :---: | :---: | :---: |
| 1 | 2 | $\mathbf{P}_{1 \mathrm{j}}=\mathrm{P}_{2 \mathrm{j}}$ |
| 0.11 | 0.18 | 0.42 |
| 0.17 | 0.16 | 0.15 |
| 0.10 | 0.11 | 0.30 |
| 0.19 | 0.09 | 0.13 |

(b) Find the number of quantities required for 2 estimating the expected return and standard deviation for 300 securities in Markouritz model and how many estimates are required for these securities while using single index sharpe model.
P.T.O.
2. Estimate the error variance for multiple regression model fit to the data given below.

| $\mathrm{X}_{1}$ | 6 | 4 | 2 | 0 | 8 | 2 | 4 | 6 | 8 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{2}$ | 10 | 9 | 5 | 2 | 15 | 6 | 7 | 11 | 13 | 7 | 8 | 9 |
| Y | 7 | 6 | 7 | 2 | 11 | 3 | 10 | 8 | 14 | 2 | 8 | 12 |

Also find a linear regression equation that best fit the given data.
3. (a) A patient arrives at the hospital after fasting with blood glucose concentration of $100 \mathrm{mg} / 100 \mathrm{ml}$ blood. The blood glucose concentrations $1 \mathrm{hr}, 2 \mathrm{hrs}$ and 3 hrs after obserbing glucose are 150, 90 and $110 \mathrm{mg} / 100 \mathrm{ml}$ blood. Discuss the health status of the patient whether the patient has normal health suffering from diabetes.
(b) Suppose the population of fishes satisfy exponential growth model with an increase by $20 \%$ in an hour. If initial population is 20000, then find the population after 5 hours. How much time is required by the population to grow to triple of its initial size?
4. The transportation costs of 600 tons of a certain type of material from four factories $\mathrm{B}_{1}, \mathrm{~B}_{2} \mathrm{~B}_{3}$, and $B_{4}$ to three target stores $T_{1}, T_{2}$ and $T_{3}$ are given in the following table :
$\mathrm{T}_{1}$
$\mathrm{~B}_{1}$
$\mathrm{~B}_{2}$
$\mathrm{~B}_{3}$
$\mathrm{~B}_{4}$$\left[\begin{array}{ccc}8 & \mathrm{~T}_{3} \\ \mathrm{~B}_{4} & 5 \\ 6 & 6 & 6 \\ 10 & 8 & 4 \\ 8 & 6 & 4\end{array}\right]$

The daily capacity of each of the factory is 150 per day and the daily requirements over each target store is 200 . Find the allocation for each factory to each target store which minimizes the total transportation cost.
5. Consider the discrete time population model given by.
$N_{t+1}=\frac{r N_{t}}{1+\left(\frac{N_{t}}{K}\right)^{b}}$, for a population $N_{t^{\prime}}$
where $K$ is the carrying capacity of the population, $r$ is the intrinsic growth rate and $b$ is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $0<r<1$. Also find the first bifurcation value of the parameter $r$.
6. (a) A tax consulting firm has four service counters in its office to receive people who have problems and complaints about their income, wealth and sales taxes. Arrivals average 80 persons in an 8 hours service day. Each tax adviser spends an irregular amount of time servicing the arrivals which are found to have an exponential distribution. The average service time is 20 minutes. Calculate the average number of customers in the system.
(b) What are residual plots and box plots? Give 2 an example of each.

