No. of Printed Pages : 7

MMT-008

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

00658

Term-End Examination

June, 2011

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 8 is* ***compulsory****. Answer any* ***six*** *questions from question number* ***1 to 7****. Use of calculator is* ***not*** *allowed.*

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1. (a) Let the random vector $\underline{X}' = (x_1\ x_2\ x_3)$ have mean vector μ_x and var-cov matrix Σ is as given below **9**

$$\mu_x = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & 9 \end{bmatrix}$$

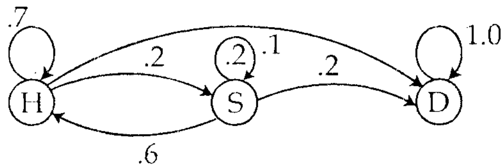
Taking x_3 dependent variable and x_1, x_2 independent variables.

(i) Fit the equation $x_3 = b_0 + b_1 x_1 + b_2 x_2$.

(ii) Obtain multiple correlation coefficient between x_3 and x_1, x_2

(iii) Find minimum mean square errors in the estimate of x_3.

(b) The transition graph of a markov chain having three states healthy, sick and dead is given below which provides transition probabilities for the changes in a week in the condition of a patient. **6**



If the probability that a patient is healthy be ·8 then ;

(i) find the probability that he/she will be sick in the coming first week.

(ii) find the probability that he/she will remain healthy in next two weeks. Also write transition probability matrix.

2. (a) In a city 20% of adults were smokers. 70% of smokers and 40% of non-smokers were found male. An adult was chosen from the city at random. **6**

(i) What is the probability that the chosen adult is male ?

(ii) If the adult was a male then what is the probability that he was a smoker ?

(iii) If the adult was a male then what is the probability that he was a non-smoker ?

(b) Arrivals at a telephone booth follow Poisson law with an average 6 per hour. Length of phone calls follow exponential distribution with mean 3 minutes. 6

(i) What is the probability that a person finds the booth empty when he/she arrives ?

(ii) What is average length of the queue ?

(iii) What will be average waiting time in the queue ?

(iv) What should be arrival rate which will make average waiting time in the system more than 3 minutes ?

(c) Let the Poisson process $\{x\ (t) : t \geq 0\}$ with intensity α be independent of another such process $\{y\ (t) : t \geq 0\}$ with intensity β. Then show that $\{z\ (t) = x\ (t) + y(t) : t \geq 0\}$ is also a Poisson process with intensity $\alpha + \beta$. 3

3. (a) State the assumptions of the Poisson process. Taxis arrive at a spot from North at a rate of 40 per hour and from south at a rate 60 per hour in accordance with independent Poisson Process. Find the probability that a person will have to wait for taxi at the spot more than 2 minutes. How many taxies will arrive at the spot in 10 minutes on the average ? 6

(b) Joint probability distribution of two random variables x_1 and x_2 are given in the following table : 9

$x_1 \backslash x_2$	0	1
−1	.16	.14
0	.04	.26
1	.10	.30

Find.

(i) mean vector

(ii) var-cov matrix

(iii) correlation matrix

4. (a) A Markov chain has following transition Matrix. 6

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1/2 & 1/2 & 0 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Determine probabilities of ultimate return to the states and mean recurrence times of the states. Check whether chain is irreducible.

(b) A random sample of size 4 from $N_2(\mu, \Sigma)$ is given below. 9

x_1	2	8	6	8
x_2	12	9	9	10

Test the hypothesis $H_0 : \mu' = [7, 11]$ against $H_1 : \mu' \neq [7, 11]$ at 5% level of significance. (you may like to use the values $F_{2,4(.05)} = 19.25$ and $F_{2,2\,(.05)} = 19.00$)

5. (a) Variables x_1, x_2 and x_3 have following var-cov matrix. **9**

$$\Sigma = \begin{bmatrix} 1 & .63 & .4 \\ .63 & 1 & .35 \\ .4 & .35 & 1 \end{bmatrix}$$

Write its factor model.

(b) Suppose in a branching process, the offspring distribution is given as. **3**

$$p_k = n_{c_k} p^k q^{n-k} \ ; 0<p<1,\ q=1-p,\ k=0, 1, 2,$$

what will be the probability of extinction of this branching process ?

(c) Suppose that families migrate to an area at a Poisson rate $\lambda = 2$ per week. The number of people in each family is independent and takes on values 1, 2, 3, 4 with respective probabilities $\frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}$. Find the expected value and variance of the number of individuals migrating to this area during a fixed five week period. **3**

6. (a) $\{x_n : n \geq 1\}$ be interoccurence times iid as exponential distribution with rate λ. Obtain distrubution of N(t), number of occurences in time t. Obtain also renewal function Mt and its Laplace transform. State the relation between $\bar{\bar{M}}_t$ and $\bar{\bar{F}}_t$, and verify the relation. **5**

(b) Let $[x_1, x_2, x_3]$ be distributed as $N_3(\mu, \Sigma)$ 6

where $\mu = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

Find :

(i) conditional densities $f(x_2/x_1, x_3)$, $f(x_1, x_2/x_3)$

(ii) the distribution of $z = 2x_1 + x_2 - x_3$.

(c) Three white and three black balls are distributed in two urns in such a way that each contains three balls. The state of the systems i is the number of white balls in the first urn, i = 0, 1, 2, 3. At each step one ball is drawn from each urn and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn. Let x_n denote the state of the system after the n^{th} step. 4

(i) Check whether $\{x_n, n = 0, 1, 2 ...\}$ is a Markov chain or not. Give reasons.

(ii) If so, then calculate its transition probability matrix and determine the limiting probabilities.

7. (a) Customers enter in a bank according to Poisson law with an average rate 21 per hour. The bank has three counters to serve them. Service time at each counter follows exponential distribution with mean time 6 minutes. 8

As an alternative, the management is thinking to install an automatic service machine which will be able to serve three times of customers than a single counter. Which of the two systems will be more efficient in terms of average time spent in the bank ?

(b) In an investigation for relationship among three variables x_1, x_2 and x_3, the sample var-cov matrix is obtained as given below. 7

$$S = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 8 & -2 \\ 3 & -2 & 5 \end{bmatrix}$$

Assume, $n = 30$ as the sample size.

(i) Find the correlation matrix.

(ii) Test the hypothesis H_0 : variables are pairwise uncorrelated against H_1 : same of the variables are correlated, at 5% level of significance.

(You may like to use the following values $\log_e 32 = -1.1394$,

$x^2_{2\,(.05)} = 5.99$, $x^2_{3\,(.05)} = 7.81$)

8. State whether following statements are true or false. Justify your answer with valid reasons. **10**

(a) If u < c and equal costs and equal prior probabilities are considered, then the corresponding observation belongs to population π_1.

(b) A set containing states of a Morkov chain that communicated with each other, will be closed ways.

(c) Traffic intensity in a queuing system should be less than unity.

(d) variance - covariance matrix of a random vector is non-negative definite.

(e) In a finite Morkov chain, all states are always recurrent.