00673

MASTER'S IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE M.Sc. (MACS)

Term-End Examination

June, 2011

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours | *Maximum Marks : 50*

Note : *Question No.* **1** *is* ***compulsory.*** *Do any* ***four*** *questions out of question nos.* **2** *to* **7**. *All computations may be kept to* **3** *decimal places. Use of calculators is* ***not*** *allowed.*

1. State whether the following statements are true or false. Justify your answer with the help of a short proof or a counter example. **2x5=10**

(a) $\cos z = J_0(z) + 2\sum_{m=0}^{\infty} (-1)^m \, J_{2m+1}(z).$

(b) For $\lambda = \frac{1}{6}$, the schmidt method for solving parabolic equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in a region R $(a \le x \le b)$ and $0 \le t \le T$ is of order $0(k^2 + h^4)$.

(c) For h > 0.0139, 4^{th} order Runge-Kutta method used to solve the initial value problem.

$$y' = -200\, y,\ y\ (0) = 1,$$

produces stable results.

(d) For the differential equation

$x^2 (x-3)^2\, y'' + 2\,(x-3)\, y' + (x+2)\, y = 0,$

$x = 3$ is an irregular singular point.

(e) The particular integral of the equation

$(D^3 - 5D^2 + 7D - 3)\, y = e^{3x}$ is $\frac{1}{4}\, x\, e^{3x}$.

2. (a) Using Laplace transforms, solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $x>0$, $t>0$ given that 5

$u\ (0, t) = 10 \sin 2t$, $u\ (x, 0) = 0$

$u_t\ (x, 0) = 0$ and $\lim_{x \to \infty} u\ (x, t) = 0.$

(b) Using central difference approximation method solve the following boundary value problem $y'' + y + 1 = 0,\ y(0) = 0,\ y(1) = 0,$ taking $h = \frac{1}{2}$. Also estimate the order of the method. 5

3. (a) Given $\frac{dy}{dx} = y - x$, where $y(0) = x$, find $y(0.1)$ and $y(0.2)$, using Runge - Kutta fourth order formula with $h = 0.1$. 4

(b) Prove that ; 3

$$J_{n-1}(x) = \frac{2}{x}\left[n J_n - (n+2) J_{n+2} + (n+4) J_{n+4} \ldots\ldots\ldots\ldots\right]$$

(c) By definition Laguerre polynomial is given by ; 3

$$L_n(t) = \sum_{r=0}^{n} \frac{(-1)^r \, \lfloor n \; t^r}{\lfloor n-r \; (\lfloor r)^2}.$$

Show that $\int_0^\infty e^{-st} L_n(t)\, dt = \frac{1}{s}\left(1 - \frac{1}{s}\right)^n$.

4. (a) Solve the boundary value problem 5

$$y'' - 3y' + 2y = 0$$

$$y(0) = 1, \; y(1) = 0$$

Using second order finite difference method with $h = \frac{1}{3}$.

(b) Solve the equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions. 5

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(1, t) = 0 \end{array}\right\} (t > 0) \text{ and } \left.\begin{array}{l} \frac{\partial u}{\partial t}(x, 0) = 0 \\ u(x, 0) = \sin^3 \pi x \end{array}\right\}$$

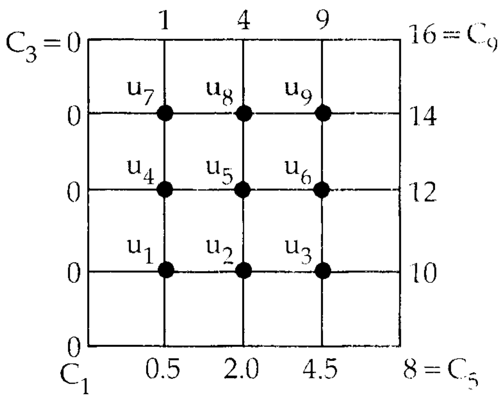
for all x in $0 \le x \le 1$,

using the explicit formula

$$u_i^{j+1} = -u_i^{j-1} + \alpha^2 (u_{i-1}^j + u_{i+1}^j) + 2(1-\alpha^2)\, u_i^j \text{ with } h = \frac{1}{4},\ k = \frac{1}{5} \text{ and } \alpha = \frac{k}{h} < 1.$$

Use the central difference approximation to the derivatives to obtain initial condition.

5. (a) Using five point difference formula solve the Laplace equation $\frac{\partial^2 u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$, where the function u satisfies at all points within the square as shown in adjacent figure and has the boundary values as indicated. Write the Gauss - Seidal iteration scheme to solve the resulting equation. 5



(b) Find the power series solution of the equation $(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - xy = 0$ in powers of x. **5**

6. (a) Find the Fourier Cosine transform of the that function $k(t) = \begin{cases} 1-|t|, & \text{for } -1 \le t \le 1 \\ 0, & \text{otherwise.} \end{cases}$ **2**

(b) Using Laplace transforms, solve **3**

$$y'' + y = \cos 2t,$$

given that $y(0) = 1$, $y'(0) = -2$,

(c) Determine the appropriate Green's function by using the method of variation of parameters for the boundary value problem **5**

$$\frac{d^2y}{dx^2} + y = e^x \sin x \cos 2x,$$

with $y'(0) >= 0$, $y(1) = 0$,

7. (a) (i) Find $L^{-1}\left(\frac{1}{\sqrt{2s+3}}\right)$, **4**

(ii) Given $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$, find $L\left\{\frac{\sin 2t}{t}\right\}$.

(b) Solve the problem **6**

$$\nabla^2 u = x + y,\ 0 \le x \le 1,\ 0 \le y \le 1$$

$$u = \frac{1}{6}(x^2 + y^2) \text{ on the boundary,}$$

Using the Galerkin's method with rectangular elements and one internal node $(h = 1/2)$.