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ET-102

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B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering)

Term-End Examination

June, 2011

ET-102 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

Note : *Answer **any ten** questions. Use of calculator is allowed.*

1. Show that the harmonic series of order p i.e. **7**

$$\Sigma\frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots \propto$$

Converges for $p>1$ and diverges for $p\leq 1$.

2. Test for convergence of the series **7**

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots\ldots \propto$$

3. Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. **7**

4. Obtain a half-range cosine series for 7

$f(x) = kx$ for $0 \le x \le l/2$,

$f(x) = k(l - x)$ for $l/2 \le x \le l$.

Also deduce the sum of the series

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$$

5. (a) Find the Laplace transform of 4+3

$e^{-3t}(2 \cos 5t - 3 \sin 5t)$.

(b) Find the inverse transform of

$$\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$$

6. Solve the following equation by the Laplace transform method. 7

$y'' + 4y' + 3y = e^{-t}$, $y'(0) = y'(0) = 1$

7. If $2 \cos \theta = x + \frac{1}{x}$ and $2 \cos \phi = y + \frac{1}{y}$, show that 7

one of the values of

$$x^m y^n + \frac{1}{x^m y^n} \text{ is } 2 \cos (m\theta + n\phi)$$

8. Show that the polar form of Cauchy - Riemann equations are 7

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$

9. Find the bilinear transformation which maps the points $z=1, i, -1$ onto the points $w=i, 0, -i$. 7

10. Prove that $\int_C \frac{dz}{z-a} = 2\pi i$ where C is the circle $/z-a/=r$. 7

11. Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2 (z+2)}$ and the residue at each pole. 7

12. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x e^x \sin x$. 7

13. Solve $(x^2-y^2-z^2)\, p + 2xy\, q = 2xz$. 7

14. Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\,\partial y}-6\,\frac{\partial^2 z}{\partial y^2} = x+y$ 7

15. Using the method of separation of variables, solve 7

$$\frac{\partial u}{\partial x}=2\,\frac{\partial u}{\partial t} + u \text{ where } u(x, 0)=6\,e^{-3x}$$