B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering)

Term-End Examination

June, 2011

04564

ET-101(B) : MATHEMATICS-II (Probability & Statistics)

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any Seven questions. All questions are of equal marks. Use of calculator is permitted.*

1. (a) Can events be : **4**

(i) mutually exclusive and exhaustive

(ii) exhaustive and independent

(iii) mutually exclusive and independent ?

Justify your answer in each case by giving an example.

(b) The probability of n independent events are $p_1, p_2, \ldots\ldots\ldots, p_n$. Find the probability that at least one of the events will happen. Using this, find the probability of obtaining at least one 6 in a throw of four dice. **6**

2. (a) Suppose an assembly plant receives its voltage regulators from three different sources, 60% from B_1, 30% from B_2 and 10% from B_3. Let 95%, 80% and 65% of the supply received respectively from the sources B_1, B_2 and B_3 perform as per specifications laid. If A is the event that a voltage regulator received at the plant performs as per specifications then find P(A). **6**

(b) For the two events A and B, prove that $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$. **4**

3. (a) Define a Poisson variate. Find its mean and variance. Describe a situation where Poisson model is applicable. **5**

(b) Two cards are drawn from a pack of 52 cards. Find the probability that draw includes an ace and a ten. **5**

4. (a) For a normal distribution prove that mean = mode = median. **4**

(b) In a production of iron rods the diameter X can be approximated to be normally distributed with mean 2 inches and S.D. 0.008 inches. **6**

(i) What percentage of defectives can we expect if we set the acceptance limit at 2 ± 0.02 inches ?

(ii) How should we set the acceptance limits to allow for 4% defectives ?

5. (a) The joint density of X and Y is given by : **6**

$$f(x,y) \begin{cases} \frac{12}{5}x(2-x-y), & 0<x<1, 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

Compute the conditional density of X, given that $Y = y$, where $0 < y < 1$.

(b) If X and Y are two independent random variables, then show that $\mathrm{Var}(aX+bY) = a^2 \,\mathrm{Var}(X) + b^2 \,\mathrm{Var}(Y)$. **4**

6. (a) Let Xi assumes the value 1 with probability p and o with probability $q=1-p$. Verify that the Week Law of large Numbers holds for the sequence of independent and identically distributed random variables Xi's **4**

(b) Suppose that the amount of weight W (in'000 pounds) that a certain span of a bridge can withstand without resulting in **6** structural damage is normally distributed with mean 400 and S.D. 40. Suppose that the weight (in'000 pounds) of a car is random variable with mean 3 and S.D. 0.3. How many cars would have to be on the bridge span for the probability of structural damage to exceed 0.1 ?

7. (a) Explain the following terms : 4

(i) Null hypothesis and alternative hypothesis.

(ii) Type I and Type II errors.

(b) Following data gives 11 measurements of the same object on the same instrument 2.7, 2.5, 2.3, 2.4, 2.3, 2.5, 2.7, 2.5, 2.6, 2.6, 2.5. At 1% level, test the hypothesis that the variance of the instrument is no more than 0.16. 6

8. (a) Let $X_1, X_2, \ldots\ldots\ldots X_n$ be a random sample from a population having a mean μ and variance σ^2. Show that ; 5

$$\overline{X} = \frac{2}{n^2}\sum_{i=1}^{n} i X_i$$

is consistent estimator of μ.

(b) The life - time T of a component has pdf $f(t) = \alpha e^{-\alpha(t-\beta)}$, $t > \beta > 0$. Based on a random sample of size n on T, find MLE of **5**

(i) α, if β is known,

(ii) β, if α is known,

9. (a) The test runs with six models of an experimental engine showed that they operated respectively for 24, 28, 21, 23, 32 and 22 minutes with a gallon of fuel. Obtain a 99% confidence interval for the average run time of engine with a gallon of fuel. **5**

(b) The following are 10 measurements on some characteristic measured by same instrument by two technicians A and B. Is B more consistent than A at 5% level of significance. **5**

A	13	15	7	15	5	12	9	3	20	11
B	12	7	2	8	6	9	5	7	6	8