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MIA-005 F2F

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M.Sc. ACTUARIAL SCIENCE

Term-End Examination

June, 2010

MIA-005 F2F : STOCHASTIC MODELING AND SURVIVAL MODELS

Time : 3 hours *Maximum Marks : 100*

Note : *In addition to this paper you should have available Actuarial Tables and your own electronic calculators.*

SECTION - A

Answer ***any five*** questions :

1. (a) How would you describe a stochastic model ? Give two advantage of stochastic model over deterministic model. **4**

(b) A markov jump process has transition probabilities given by Pij(t).

(i) Set out the formula for the transition rates σij(t) in terms of the transition probability Pij(t) clearly state any assumption that you make. **2**

(ii) Show that the sum of transition rates out of any state is zero. **2**

2. (a) Explain the terms " Undergraduation" and "Overgraduation". 3

(b) List the possible dangers to a life company of using Undergraduated or Overgraduated mortality rates. 5

3. A large life office is investigating the recent mortality experience of its term assurance policyholders. It has been decided to graduate the data by reference to a standard table using the formula :

$$\frac{q_x}{q_s} = ax + b.$$

where q_x^s is the rate for the standard table.

(a) Outline the considerations that you would take into account in choosing an appropriate standard table. 5

(b) Explain how you would check whether the above formula was suitable. 3

4. A No-claim discount system operated by a motor insurer has the following four levels.

Level 1 : 0% discount

Level 2 : 25% discount

Level 3 : 35% discount

Level 4 : 50% discount

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The rules for moving between these levels are as follows :

- following a year with no claims, moves to the next higher level, or remain at level 4.
- following a year with one claim, move to the next lower level or remain at level 1.
- following a year with two or more claims, move back two levels, or move to level 1 (from level 2) or remain at level 1.

For a given policyholder the probability of no claims in a given year is 0.85 and the probability of making one claim is 0.12,

(a) Write down the transition matrix of this chain. 1

(b) Explain whether the chain is irreducible and/or a periodic. 2

(c) Calculate the long-run probability that a policyholder is in discount level 2. 5

5. During a period of length T years, you observe a total of N lives between the ages of x and $x+1$. You do not necessarily observe each life for the entire year of age. The total time spent under observation by the N lives is V. d deaths are observed.

(a) State the assumptions underlying the poisson model for d, given that the force of mortality between the ages of x and $x+1$ is a constant, μ. 2

(b) Show that the maximum likelihood estimator of μ is D/V, under the assumption in (a) above. **4**

(c) Show that the maximum likelihood estimator has : **2**

(i) an expected value of μ.

(ii) a variance of μ/V.

6. (a) List the data needed for the exact calculation of a central exposed to risk depending on age. **2**

An investigation studied the mortality of person aged between exact ages 40 and 41 years. The investigation began on 1^{st} Jan. 2008 and ended on 31^{st} Dec. 2008. The following tables gives details of 10 lives involved in the investigation.

Life	Date of 40^{th} Birthday	Date of Death
1.	1 March 2007	-
2.	1 May 2007	1 October 2008
3.	1 July 2007	-
4.	1 October 2007	-
5.	1 December 2007	1 February 2008
6.	1 February 2008	-
7.	1 April 2008	-
8.	1 June 2008	1 November 2008
9.	1 August 2008	-
10.	1 December 2008	-

Persons with no date of death given were still alive when the investigation ended.

(b) Calculate a central exposed to risk using the data for the 10 lives in the sample. **4**

(c) (i) Calculate the maximum likelihood estimate of the hazard of death at age 40 last birthday. **1**

(ii) Hence, or otherwise, estimate q_{40}. **1**

7. (a) State the Markov property. **1**

A stochastic process X (t) operates with state space S.

(b) Prove that if the process has independent increments if satisfies the Markov property. **3**

(c) (i) Describe the differences between a Markov chain and a Markov jump process. **1**

(ii) Explain what is meant by a Markov chain being irreducible. **1**

An actuarial student can see the office lift (elevator) from his desk. The lift has an indicator which displays on which of the office's five floors it is at any point in time. For light relief the student decides to construct a model to predict the movements of the lift.

(iii) Explain whether it would be appropriate to select a model which is :

(A) irreducible. **1**

(B) has the Markov property. **1**

SECTION - B

Answer ***any four*** questions :

8. (a) Explain the differences between random censoring and type 1 censoring in the context of an investigation into the mortality of life insurance policyholders. 3

(b) Define Type I and Type II censoring. 2

(c) A employer has carried out a mortality investigation of its employers for making provision for the gratuity benefits. It observed a sample of independent employees aged between 55 and 60 years. Employees were followed from their 55^{th} birthday until either they died, or they withdrew from the investigation or they retired while still alive (whichever of these events occurred first). The retirement age is 60^{th} birthday.

(i) Describe the types of censoring that are present in this investigating. 2

(ii) An extract from the data for 20 employees is shown in the table below. Use these data to calculate the Kaplan-Meier estimate of the survival function 8

Person number	Last age at which person was observed (years and months)		Outcome
1	55	6	Died
2	55	6	Withdrew
3	56	0	Died
4	56	0	Died
5	56	6	Withdrew
6	57	3	Died
7	57	3	Withdrew
8	57	3	Died
9	57	6	Withdrew
10	58	0	Withdrew
11	58	3	Died
12	58	3	Withdrew
13	59	3	Withdrew
14	59	6	Withdrew
15	59	9	Died
16	60	0	Retired
17	60	0	Retired
18	60	0	Retired
19	60	0	Retired
20	60	0	Retired

9. Using COX Regression Model, a study has been carried out on TB patients as to how certain factors affects the future lifetime after a person contacts Tuberculosis. The survival and smoking habits are shown in the table below. Patients have been labelled as "Censored", if they were still alive at the end of the investigation or if their death was not considered to be attributable to the TB.

Patient numbers	Time of death	Smoker (Yes/no)	Censored
1	12	Yes	No
2	Still alive	No	Yes
3	20	No	No
4	22	Yes	Yes
5	19	No	Yes
6	18	No	No

(a) How you would use the COX Model in this study, assuming that smoking status was the only covariate ? Your answer should include a description of the relevant Cox Model and you should define any notation that you use. **4**

(b) Using the convention that $Z=1$ for smokers and $Z=0$ for non-smokers, write down the partial likelihood for these data. Simplify your expression as far as possible. **4**

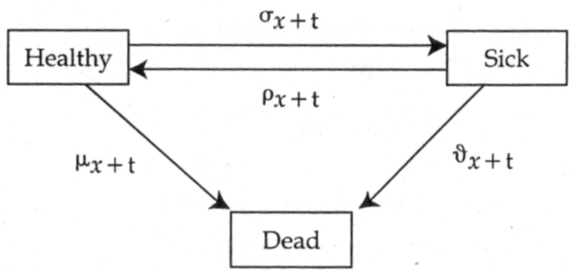
(c) Calculate the maximum partial likelihood estimate of the model parameter. **4**

(d) You are now given the following additional data : **3**

Patient number	Time of death	Smoker (yes/no)	Censored
7	12	Yes	No
8	Still alive	No	Yes

Write down the partial likelihood using the data from all 8 patients.

10. A three-state Markov model is represented by the following transition diagram :



The symbols σ_{x+t}, ρ_{x+t}, μ_{x+t} and ϑ_{x+t}, represent the forces of transition at age $x+t$, where x is an integer and $0 \leq t < 1$. The symbol ${}_tp_x^{\overline{ii}}$, $i = H, S, D$, represents the probability that a life, who is in state i at age x, remains in state i until at least age $x+t$.

(a) Write down the assumptions that are usually made when applying this model. 2

(b) Derive the differential equation : 5

$$\frac{\partial}{\partial t}\, {}_tp_x^{\overline{HH}} = -\, {}_tp_x^{\overline{HH}} \left(\sigma_{x+t} + \mu_{x+t}\right)$$

and write down the relevant initial condition.

(c) In a mortality and morbidity investigation the following data were recorded for lives between exact ages 50 and 51 : 8

Total Time (in years) spent in the healthy state :	750
Total Time (in years) spent in the sick state :	32
Number of transitions from the healthy state to the sick state :	15
Number of transitions from the sick state to the healthy state :	35
Number of transitions from the healthy state to the dead state :	3
Number of transitions from the sick state to the dead state :	5

(i) Write down the likelihood function and derive the maximum likelihood estimate of μ_{50+f}. State the value of f.

(ii) Construct an approximate 90% confidence interval for μ_{50+f}.

(iii) Estimate the value of $P_{50}^{\overline{HH}}$.

11. (a) (i) Define Poisson process. 2

(ii) Sum of two independent Poisson process is a Poison process. 3

(b) Given a stochastic process pij $(s_1 t)$ which is the probability of going from state "i" to state "j" between time "s" to time "t".

(i) Write down the integrated form of Kolomogorov backward and forward equations when :

(A) $i \neq j$ **3**

(B) $i = j$ **1**

(ii) Derive the integrated form of the forward equations when : **6**

(A) $i \neq j$

(B) $i = j$

12. (a) Explain the meaning of the rates of mortality usually denoted q_x and m_x and the relationship between them. **3**

(b) Show that m_x can be expressed as a weighted average of the force of mortality at each age between x and $x+1$ and state what the weights are. **2**

(c) In a certain population, $q_x = 0.4$. Calculate the value of m_x assuming : **7**

(i) that deaths are uniformly distributed between the ages of x and $x+1$.

(ii) a constant force of mortality between ages of x and $x+1$.

(iii) that the Balducci assumption holds between the ages of x and $x+1$.

(d) Comment on your results in part (c) by considering the force of mortality over the year of age x to $x+1$ in each case. **3**

13. The mortality experience of some whole life assurance policyholders has been compared with a standard mortality table for assured lives. The following is an extract from the data.

Age (x)	Actual deaths θ_x	Expected deaths $Ex\ q_x^s$	$\theta x - q_x^s$
60	37	42.88	–5.88
61	40	61.73	–21.73
62	28	38.06	–10.06
63	41	47.23	–6.23
64	34	40.36	–6.36
65	40	49.98	–9.98
66	27	25.13	1.87
67	15	22.25	–7.25
68	16	26.23	–10.23
69	30	27.61	2.39
70	23	25.11	–2.11
Total	331	406.56	–75.57

(a) Carryout a comparison between the actual and expected mortality experience, using the following statistical tests : 12

(i) Chi-squared test.

(ii) Cumulative deviations test.

(iii) Serial correlations test.

You should state the appropriate null hypothesis and, for each test, the conclusion reached with regards to this hypothesis.

(b) Summarise what you can infer about the mortality experience of these policyholders from your analysis, giving your reasons. 3