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MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

00575

Term-End Examination

June, 2010

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

*Note : **Question No. 1** is **compulsory**. Do **any four** questions from question number 2 to 7. Use of calculator is **not** allowed.*

1. (a) (i) Define the weight enumerator of a code. **6**

(ii) Find the weight enumerator polynomial of the code

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

(b) Define the q-cyclotomic coset of s modulo $(q^t - 1)$. Compute the 2-cyclotomic cosets modulo 7. **4**

2. (a) Let $G = \begin{pmatrix} 1 & 0 & 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 & 1 & 2 \end{pmatrix}$ be a generator matrix for the ternary linear code C. 7

(i) Write the generator matrix in the standard form and hence find the parity matrix.

(ii) Write the generator and parity matrix of the dual code. Is the code self-dual ? Justify your answer.

(b) Prove that a self-dual code has even length n and dimension $\frac{n}{2}$. 3

3. (a) Show that the distance function is a metric. 5

(b) Let r be an integer with $0 \leq r \leq m$. If $0 \leq r < m$, prove that $R(r, m)^{\perp} = R(m-r-1, m)$. 5

4. (a) Let $g(x) = 1 + x + x^3$ be the generator polynomial of a [7, 4] cyclic code. Write its generator matrix and parity check matrix. 5

(b) If $n = \frac{q^r - 1}{q - 1}$, where gcd $(r, q-1) = 1$, let C be the narrow-sense BCH code with defining set $T = C_1$ (cyclotomic set). Show that C is the Hamming Code $H_{q, r}$. 5

5. (a) Let C be the [15, 7] narrow-sense binary BCH code of designed distance $\delta = 5$, which has defining set $T = \{1, 2, 3, 4, 6, 8, 9, 12\}$. Using the primitive 15th root of unity α, $\alpha^4 = \alpha + 1$ the generator polynomial of C is $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. If $y(x) = 1 + x + x^5 + x^6 + x^9 + x^{10}$ is received, find the transmitted code word. 5

(b) Define convolutional codes. Give an example of a convolutional code. 2

(c) Define primitive polynomial. Give a primitive polynomial of degree 3 with justification. 3

6. (a) Let C be (4, 2) convolutional code with generator matrix. 6

$$G = \begin{bmatrix} 1 & 1 + D + D^2 & 1 + D^2 & 1 + D \\ 0 & 1 + D & D & 1 \end{bmatrix}$$

Use elementary row operations to find two more generator matrices for C.

(b) Show that the binary odd-like Quadratic Residue codes of length 23 are the [23, 12, 7] binary Golay code. 4

7. (a) (i) Define Gray map $G : \mathbb{Z}_4 \to \mathbb{F}_2^{\,2}$. 5

(ii) Let C = {0000, 1113, 2222, 3331, 0202, 1313, 2020, 3131, 0022, 1131, 2200, 3313, 0220, 1333, 2002, 3111} be the $\mathbb{Z}_4$-linear code. Find the Gray image of C.

(b) State the Message Passing Decoding algorithm. 5