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MMT-009

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M.Sc. Mathematics with Applications in Computer Science (MACS)

Term-End Examination

June, 2010

MMT-009 : MATHEMATICAL MODELLING

Time : 1½ hours *Maximum Marks : 25*

Note : *Do **any five** questions. Use of calculator is **not** allowed.*

1. (a) In a population of birds, the proportionate birth rate and the proportionate death rate are both constant, being 0.45 per year and 0.65 per year, respectively. Formulate a model of the population and discuss its long term behaviour. **2**

(b) Let P(t), measured in kg, be the total mass or biomass of the fish population in a pond at time t. Write the continuous model for the population growth using logistic equation. The intrinsic growth rate r and the carrying capacity k are given the values 0.71 per year and 80.5×10^6 kg, respectively. If the initial biomass is $P_0 = 0.25$ k, find the biomass after 2 years. Also find the time t_1 for which $P(t_1) = 0.75$ k. **3**

2. (a) Explain how a portfolio along the efficient frontier is better than other portfolios in the feasible set. 3

(b) A simple model including the seasonal change that affects the growth rate of a population is given by $\frac{dx}{dt} = C\,x\,(t)$ cost. Where C is a constant. If x_0 is the initial population, solve the equation and determine the maximum and minimum populations. 2

3. Consider the discrete population model governed by $N_{t+1} = N_t \exp\left[r\left(1-\frac{N_t}{k}\right)\right]$ where r and k are positive constants. Find out the steady states and do the stability analysis corresponding to each steady state. 5

4. (a) Give one example each from the real world for the following along with justification for your example : 3

(i) A non-linear model

(ii) A stochastic model

(iii) A linear, deterministic model

(b) The control parameter of growth and decay of a tumour are, respectively, 1000 and 500 per day. Also, the damaged cells migrate due to vascularization of blood at the rate of 200 cells per day. Find the ratio of the number of tumour cells after 50 days with the initial number of tumour cells. 2

5. (a) Consider the following data showing observations on the quantity demanded of a certain commodity depending on commodity price and consumer's income. 3

Quantity demanded	Price (in Rs.)	Income (in Rs.)
100	5	1000
75	7	600
80	6	1200
70	6	500
50	8	300

(i) Find a linear regression equation that best fit the data.

(ii) Obtain the coefficient of multiple determination for the data.

(b) Find the number of covariances needed for an evaluation of 200 securities using the Markowitz model. Also calculate the total number of pieces of information needed. 2

6. A company has three factories F_1, F_2, F_3 that supply to three markets M_1, M_2, M_3. The transportation costs from each factory to each market are given in the table. Capacities 'ai's' of the factories and market requirements 'bj's' are shown below. Find the minimum transportation cost. 5

	M_1	M_2	M_3	a_i
F_1	2	1	3	20
F_2	1	2	3	30
F_3	2	1	2	10
b_j	10	10	20	40/60