No. of Printed Pages : 3

	L
MMT-005	l
	l

## MASTER'S IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE M.Sc. 0 (MACS) 0 Term-End Examination 0 June, 2010 MMT-005 : COMPLEX ANALYSIS Time : 1½ hours Maximum Marks : 25 Note : Question No. 1 is compulsory . Attempt any three

- **Note**: Question No. 1 is compulsory. Attempt any three questions from question number 2 to 5. Use of calculator is **not** allowed.
- State with reasons whether the following statements are *true* or *false*: 2x5=10
  - (a) Geometrically the set of all complex numbers z such that  $z + \overline{z} = |z|^2$  represents a circle.
  - (b) Let  $s = \{z : |z| < 1 \text{ or } |z-1| < 1\}$  then *s* is a domain.
  - (c) f(z) = sinz is bounded in the complex plane.
  - (d) The zeros of  $cos\left(\frac{1}{z}\right)$  are isolated.
  - (e) The radius of convergence of the series

1

$$\sum_{n=1}^{\infty} 2^n z^{n!} \text{ is } 1$$

**MMT-005** 

**P.T.O.** 

2. (a) Show that the following function is not 2 continuous at z=0.

$$f(z) = \begin{cases} \frac{\text{Re}(z)}{|z|}, & z \neq 0\\ 0, & z = 0 \end{cases}$$

(b) Show that  $u(x, y) = x^2 - y^2$  is harmonic in **3** the whole plane. Find its harmonic conjugate and hence the associated analytic function.

3. (a) Evaluate 
$$\int_{C} \frac{4}{(4z^2 - 1)} dz$$
; 2

where C is the unit circle with Centre 1.

- (b) Prove that the transformation  $w = \frac{1}{z}$  3 transforms circles and lines into circles and lines. In particular, find the transformation of the line y=1 under the above transformation.
- 4. (a) Discuss the nature of singularities of 2  $f(z) = \frac{1}{z^3 \ sinz}.$

(b) Show that 
$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} \, \mathrm{d}x = \frac{\pi}{\sqrt{2}}$$
. 3

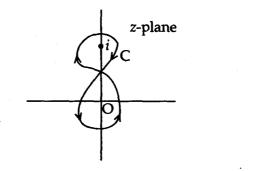
MMT-005

2

5. (a) Prove that  $Q(z) = \frac{2z-1}{2-z}$  is a conformal 3 map which maps the closed unit disk  $\overline{D} = \{z : |z| \le 1\}$  onto itself.

(b) Evaluate  $\int_{C} \frac{e^{z}}{(z-i)^{3}} dz$ , where C is the 2

eight-like figure given below :



3