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B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering)

Term-End Examination

June, 2010

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is permitted*

1. Answer *any five* of the following : **5x4=20**

(a) Draw the graph of the function

$$f(x) = \begin{cases} -1 + x^2, & -1 \le x < 0 \\ x, & 0 \le x < 1 \\ 2, & x = 1 \\ -x + 2, & 1 < x < 2 \\ 0, & 2 < x \le 3 \end{cases}$$

and discuss its continuity on $[-1, 3]$

(b) Discuss the differentiability and continuity of the function $y = 3|x| + 1$ at $x = 0$. Also plot its graph on $[-1, 1]$

(c) Find where the tangent is parallel to the x-axis for the curve $y^3 = x^2(2 - x)$.

(d) If $f(x) = x^2 - \dfrac{1}{x^2}$, prove that $f(x) = -f\left(\dfrac{1}{x}\right)$.

(e) Expand $\tan\left(x + \frac{\pi}{4}\right)$ as far as the term x^4.

(f) Find the diameter and height of a cylinder of maximum volume which can be cut from a sphere of radius 12 cm.

(g) If $u = \sqrt{yz}$, $v = \sqrt{zx}$, $w = \sqrt{xy}$ and $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\phi$, then show that

$$\frac{\partial(u, v, w)}{\partial(r, \theta, \phi)} = \frac{1}{4} r^2 \sin\phi$$

2. Answer *any four* of the following : **4x4=16**

(a) Evaluate $\int x^2 . e^x \, dx$.

(b) Show that the area of the region that lies inside the circle $r = a\cos\theta$ and outside the cardiod $r = a(1 - \cos\theta)$ is $\frac{a^2}{3}(3\sqrt{3} - \pi)$ sq.unit.

(c) Evaluate $\int_0^{\pi/3} \sqrt{\left(1 - \frac{1}{3}\sin x\right)} \, dx$ correct to three decimal places using Simpson's rule with 6 intervals.

(d) Find the arc length of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$.

(e) Solve the differential equation :
$$\frac{dy}{dx} + \frac{2}{x}y = 3$$

(f) Solve $\cos x \frac{dy}{dx} + y = \sin x$, $y(0) = 2$.

3. Answer *any four* of the following : **4x4=16**

(a) (i) Prove that

$$\hat{i} \times \left(\vec{a} \times \hat{i}\right) + \hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}$$

for any arbitrary vector $\vec{a}$.

(ii) Show that

$$\left(\vec{a} + \vec{b}\right) \cdot \left[\left(\vec{b} + \vec{c}\right) \times \left(\vec{c} + \vec{a}\right)\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$$

(b) Find the directional derivative of $4xz^3 - 3x^2y^2z^2$ at $(2, -1, 2)$ along the z-axis.

(c) Show that the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. Find a scalar potential u such that $\vec{F} = \text{g rad u}$.

(d) If $\vec{A} = xy\hat{i} - z\hat{j} + x^2\hat{k}$ and C is the curve $x = t^2,\ y = 2t,\ z = t^3$ from $t = 0$ to $t = 1$, evaluate $\int_C \vec{A} \times d\vec{r}$.

(e) $\vec{A} = (1, 1, 1)$ and $\vec{C} = (0, 1, -1)$ are two given vectors. Then prove that a vector $\vec{B}$ satisfying the equations $\left(\vec{A} \times \vec{B}\right) = \vec{C}$ and $\vec{A}.\vec{B} = 3$ is $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$.

(f) Use Green theorem to evaluate

$$\oint_C \left[\left(x^2 + xy\right) dx + \left(x^2 + y^2\right) dy\right]$$

where C is the boundary of the square $y = \pm 1,\ x = \pm 1$.

4. Answer *any six* of the following : **6x3=18**

(a) Check for the consistency of the system of equations.

$$2x - y + z = 4$$

$$x + 2y - 3z = 8$$

$$4x + 3y - 5z = 10$$

(b) Prove that the matrix given below is orthogonal :

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(c) Show that

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$$

where the capital letters denote the cofactor of the corresponding small letters.

(d) Define the rank of a matrix. Show that the rank of the product of two matrices cannot exceed the rank of either matrix.

(e) Determine values of 'a' for which the following system of linear equations has

(i) a unique solution

(ii) infinite number of solutions

$$x + y + z = 2$$

$$x + 2y + z = -2$$

$$x + y + (a-5)z = a - 4$$

(f) Find the eigen values and the eigen vectors of the matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(g) Prove that a matrix is singular if, and only if one of its eigen values is zero.

(h) For $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $A^n = A^{n-2} + A^2 - I$, for $n \geq 3$.

(i) The eigen vectors of a 3×3 matrix A corresponding to the eigen values 1, 2, 3 are $[-1, -1, 1]^T$, $[0, 1, 0]^T$ and $[0, -1, 1]^T$ respectively. Find the matrix A.