03162

# MCA (Revised)

## Term-End Examination

## June, 2010

## MCSE-004 : NUMERICAL AND STATISTICAL COMPUTING

*Time : 3 hours* *Maximum Marks : 100*

***Note :*** *Question number **1** is **compulsory.** Attempt **any three** questions from the rest. Use of calculator is allowed.*

**1.** (a) Estimate the relative error in $z = x - y$ when $x = 0.1234 \times 10^4$ and $y = 0.1232 \times 10^4$ as stored in a system with four-digit mantissa. **6**

(b) Show that the series $e^x = 1 + x + \frac{x^2}{2!} + .....$ becomes unstable when $x = -10$. **5**

(c) Find the root of the equation $x^x + x - 4 = 0$ using the Newton-Raphson method correct to four decimal places. **6**

(d) The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the independent variable. What is the best estimate you can give of the value of the function at the position 6 of the independent variable. Apply Lagrange's formula. **7**

(e) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface : 8

| $x$ = height | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|
| $y$ = distance | 10.63 | 13.03 | 15.04 | 16.81 | 18.42 | 19.90 | 21.27 |

Find the value of $y$ when $x = 410$ using Newton's Backward Interpolation formula.

(f) Five men in a group of 20 are graduates. If 3 men are picked out of 20 at random (i) what is the probability that all are graduates and (ii) what is the probability of at least one being graduate ? 8

2. (a) Find the root of the equation $x\, e^x = cos\ x$ using the secant method correct to four decimal places. 7

(b) Evaluate $\int_1^2 log\ x$ by Trapezoidal rule. 6

(c) A book contains 100 misprints distributed randomly throughout its 100 pages. What is the probability that a page observed at random contains atleast two misprints. 7

3. (a) Solve the system of equations : 10

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

Using Jacobi iteration method.

(b) Use Euler method to solve numerically the initial value problem. **10**

$v' = -2t\ v^2,\ v\ (0) = 1$

with $h = 0.2$ and 0.1 on the interval [0, 1].

**OR**

A sample of 100 dry battery cells tested to find the length of life produced the following results : **10**

$\overline{X} = 12$ hours, $\sigma = 3$ hours

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life :

(i) More than 15 hours

(ii) Between 10 and 14 hours

| Given | Z : | 2.5 | 2 | 1 | 0.67 |
|---|---|---|---|---|---|
| | Area : | 0.4938 | 0.4772 | 0.3413 | 0.2487 |

4. (a) Show that the LU decomposition method fails to solve the system of equations : **10**

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

Exact solution is $x_1 = 1$, $x_2 = 0$, $x_3 = -1$.

**OR**

Apply Runge-Kutta method to find approximate value of $y$ for $x = 0.2$, in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ where $x = 0$.

(b) A problem in statistics is given to five students A, B, C, D and E. Their chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$. What is the probability that the problem will be solved ? **10**

5. (a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$. **6**

(b) With the help of Newton's forward difference interpolation formula obtain the interpolating polynomial satisfying the data. **7**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 26 | 18 | 4 | 1 |

If a point $x = 5, f(x) = 26$, is added to above data, will the interpolation polynomial change ? Explain.

(c) What is a random variable ? Write down the expression which define Binomial, Poisson and Normal probability distribution. Give two physical situation illustrating a poisson random variable. **7**