No. of Printed Pages : **15** **MIA-010 (F2F)**

00129

M.Sc. ACTUARIAL SCIENCE

Term-End Examination

December, 2011

MIA-010 (F2F) : STATISTICAL METHOD

Time : 3 hours *Maximum Marks : 100*

SECTION - A

Attempt *any five* questions.

1. You are using simulation to find the mean and variance of a random variable that has the density function.

$$fx(x) = \frac{k}{1+ e^{-x}} \quad > \quad -1 < x < 1.$$

(a) Determine the value of the constant k. **2**

(b) Generate five pseudo - random values from this distribution based on the u (0, 1) values 0.017, 0.757, 0.848, 0.531 and 0.321 **3**

(c) You have generated 1000 such numbers and found that $\Sigma x = 165.681$ and $\Sigma x^2 = 339.275$. **3**

Use these statistics to find a 95% confidence interval for the mean of the distribution and a point estimate for the standard deviation.

2. The total claim arising from certain portfolio of insurance policies over a given month is represented by : 8

$$S = \begin{cases} \Sigma x_i & \text{if} \quad N > 0 \\ 0 & \text{if} \quad N = 0 \end{cases}$$

where N has a poisson distribution with mean 2 and $X_1, X_2, \ldots X_N$ is a sequence of iid random variables that are also independent of N.

The distribution function of x such that

$$P(x_i = 1) = \frac{1}{3} \text{ and } P(x_i = 2) = \frac{2}{3}$$

For every $i = 1 - N$

An aggregate reinsurance contract has been arranged such that the amount paid by the reinsurer is $S - 3$ (if $S > 3$) and zero otherwise. The aggregate claim paid by the direct insurer and reinsurer are denoted by S_I and S_R respectively calculate $E(S_I)$ and $E(S_R)$.

3. The number of claims on a portfolio of washing machine insurance policies follows a poisson distribution with parameter 50. Individual claim amounts for repair is a random variable 100x where x has a distribution with probability density function :

$$fx(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) \,;\, 1 < x < 5 \\ 0 \quad ; \text{otherwise} \end{cases}$$

In addition for each claim [(independent) of the cost of the repair] there is a 30% chance that an additional fixed amount of Rs. 200 will be payable in respect of water damage

(a) Calculate the mean and variance of the total individual claim amounts. 5

(b) Calculate the mean and variance of the aggregate claims on the portfolio. 3

4. (a) Claims occur on a portfolio of insurance policies according to a poisson process at a rate λ. All claims are for a fixed amount d and premium are received continuously. The insurer initial surplus is u (<d) and the annual premium income is 1.2λd. Show that the probability that ruin occurs at the first claim is : 4

$$1 - e^{\frac{-1}{1.2}\left(1 - \frac{u}{d}\right)}$$

(b) The number of claims in a year from individual policies in a portfolio is believed to follow a poisson (λ) distribution. A Gamma (5, 2) distribution is chosen as a prior distribution for λ. A random sample of 10 policies is observed , and the numbers of claims are found to be as follows : 4

4, 1, 0, 0, 2, 0, 0, 1, 3, 1

Derive the posterior distribution for λ and obtain a point estimate for λ :

Using a zero - one loss function.

5. An insurer insures a single building. The probability of a claim on a given day is p independently from day to day. Premiums of 1 unit are payable on a daily basis at the start of each day. The claim size is independent of the time of the claim and follows an exponential distribution with mean $\frac{1}{\lambda}$. The insurer has a surplus of u at time zero.

(a) Derive an expression for the probability that the first claim results in the ruin of the insurer. 6

(b) If $p = 0.01$ and $\lambda = 0.0125$ find how large U must be so that probability that the first claim causes ruin is less than 1%. 2

6. The Gamma distribution with mean μ and variance μ^2/α has density function :

$$f_Y(y) = \frac{\alpha^\alpha y^{\alpha-1} e^{-\frac{\alpha}{\mu}y}}{\mu^\alpha \Gamma\alpha} ; \ y > 0$$

(a) Show that this may be written in the form of an exponential family. 4

(b) Use the properties of exponential families to confirm that the mean and variance of the distribution are μ and $\frac{\mu^2}{\alpha}$. 4

7. The aggregate claims from a portfolio of insurance policies are $x_1, x_2, \ldots x_n$ for the year 1, 2, ...n respectively. The aggregate claim x_{n+1} for the year n+1 has to be estimated. It is known that given a fixed value of the random parameter θ, the claims $x_1, x_2, \ldots x_{n+1}$ are conditionally independent and normally distributed with mean θ and variance θ^2. The prior distribution of the parameter θ is exponential with mean μ.

(a) Obtain the unconditional mean of x_{n+1}. 1

(b) Obtain the unconditional variance of x_{n+1}. 2

(c) Find an estimate of θ which is a linear function of $x_1, x_2, \ldots x_n$ and μ and minimizes the mean squared difference between θ and the linear estimate. 5

SECTION - B

Attempt *any four* questions.

8. (a) Consider the time series model

$Y_t = Y_{t-1} + 0.5\ Y_{t-2} - 0.5\ Y_{t-3} + Z_t + 0.3\ Z_{t-1}$

Where $\{Z_t\}$ is a sequence of uncorrelated random variables each having the normal distribution with mean zero and variance σ^2.

(i) Show that the above model is a special case of the ARIMA (p,d,q) model and determine p, d, q. **2**

(ii) Let $X = (1 - B)^d Y$. Determine whether $\{X_t\}$ is a stationary time series. **2**

(iii) Calculate the autocorrelation function of $\{X_t\}$. **6**

(b) Consider the stationary autoregressive process of order 1 which is given by :

$Y_t = \alpha Y_{t-1} + Zt \quad ; 1\alpha 1 < 1$

Where {Zt} denotes white noise process with mean zero and variance σ^2 express Y_t in the form of :

$$Y_t = \sum_{j=o}^{\infty} \alpha j\ Z_{t-j}$$

and hence or otherwise find an expression for var(Y_t). The process variance in terms of α and σ^2. **5**

9. (a) The general form of a run - off triangle can be expressed as : 5

Accident year	Development Year					
	0	1	2	3	4	5
0	C 0, 0	C 0, 1	C 0, 2	C 0, 3	C 0, 4	C 0, 5
1	C 1, 0	C 1, 1	C 1, 2	C 1, 3	C 1, 4	
2	C 2, 0	C 2, 1	C 2, 2	C 2, 3		
3	C 3, 0	C 3, 1	C 3, 2			
4	C4, 0	C 4, 1				
5	C 5, 0					

Define a model for each entry C_{ij}, in general terms and explain each element of the formula.

(b) The run - off triangle given below relates to a portfolio of motorcycle insurance policies. 10

The cost of claims paid during each year is given in the table below :

(figures in Rs. 000)

(Dy) (αy)	Development Year			
Accident year	0	1	2	3
2002	2905	535	199	56
2003	3315	578	159	
2004	3814	693		
2005	4723			

The corresponding number of settled claims is as follows :

Accident year	Development Year			
	0	1	2	3
2002	430	51	24	7
2003	465	58	24	
2004	501	59		
2005	539			

Calculate the outstanding claims reserve for this portfolio using the average cost per claim method with grossing - up factors and state the assumptions underlying your result.

10. (a) The loss function under a decision problem is given by :

	θ_1	θ_2	θ_3
D_1	11	9	19
D_2	10	13	17
D_3	7	13	10
D_4	16	5	13

(i) State which decision can be discounted immediately and why ? **2**

(ii) Explain what is meant by minimax criterion and determine the minimax solution in this case. 3

(b) The random variable w has a binomial distribution such that

$$P(w = w) = \binom{n}{w} \mu^{w} (1 - \mu)^{n-w} ;$$

$w = 0, 1, 2, \ldots n$

let $Y = \frac{w}{n}$.

(i) Write down an expression for $p(Y = y)$ 1

for $y = 0, \frac{1}{n}, \frac{2}{n}, \ldots 1.$

(ii) Express the distribution of Y as an exponential family and identify the natural parameter and the dispersion parameter. 3

(iii) Derive an expression for the variance function of Y. 3

(iv) For a set of n independent observations of Y. Derive an expression of the scaled deviance. 3

11. (a) State the markov property and explain briefly whether the following processes are markov 5

(i) AR(y)

(ii) ARMA (1, 1)

(b) Show that, given a random sample of size n from log N (μ, σ^2) distribution, if an uninformative prior is used for μ, then the posterior distribution for μ is of the form **5**

$$N\left(\frac{1}{n}\sum_{i=1}^{n}\log xi\ ,\ \frac{\sigma^2}{n}\right)$$

(c) Let N be a random variable representing the number of claims arising from a portfolio of insurance policies. Let X_i denote the size of the i^{th} claim and suppose that $X_1, X_2, \ldots\ldots$ are independent and identically distributed random variables, all having the same distribution as X. The claim sizes are independent of the number of claims. **3**

Let $S = X_1 + X_2 + \ldots\ldots\ldots X_N$ denote the total claim size. Show that :

$Ms(t) = MN\ (\log M \times (t))$

(d) Suppose that N has a type 2 Negative Binomial distribution with parameters $k > 0$ and $0 < p < 1$. That is : **2**

$$P(N = x) = \frac{\sqrt{k + x}}{\sqrt{x + 1}\sqrt{k}}\ P^k\ \rho^x\ ;\ x = 0, 1, 2, \ldots\ldots$$

Suppose that x has an exponential distribution with mean $\frac{1}{\lambda}$. Derive an expression for Ms(t).

12\. (a) (i) Show that the relationship between the mode and mean (i.e. which is bigger) for a weibull distribution is dependent on the value of just one of the parameter. 3

(ii) Show that the critical value of this parameter is between 3 and 4. 2

[You are given that $\Gamma\left(\frac{3}{4}\right) = 0.893$ and

$\Gamma\left(\frac{5}{4}\right) = 0.906$.

(b) Explain why the following distribution can never belong to the exponential family. 3

(i) Continous uniform distribution in the interval $(0, 2\mu)$

(ii) Pareto with density function

$$\frac{\alpha\lambda^{\alpha}}{(\lambda + x)^{\alpha+1}} \; ; \; x > 0$$

(iii) Negative Binomial Type 2 with probability density function

$$\binom{k + x - 1}{x}(1-q)^{k}\, q^{x} \; ; \; x = 0, 1, 2 \ldots$$

$k \neq 1$

(c) The length of time taken to deal with each of n reports are independent exponentially distributed random variables with mean $\frac{1}{\lambda}$ show that the gamma distribution is conjucate prior for this exponential distribution. 5

(d) The loss function under a decision problem is given by : 2

	H_1	H_2	H_3
D_1	23	34	16
D_2	30	19	18
D_3	23	27	20
D_4	32	19	19

(i) State which decision can be discounted immediately and why ?

(ii) If H is distributed as

$P(H_1) = 0.25$

$P(H_2) = 0.15$

and $P(H_3) = 0.60$

Determine the solution based on Bayes criterion to the problem.

13. The number X of claims on a given insurance policy over one year has probability distribution given by $p(X=x)=\theta^x (1-\theta)$; $x=0, 1, 2, ...$ where θ is an unknown parameter with $0<\theta,1$.

Independent observations $x_1, x_2, ... x_n$ are available for the number of claims in the previous n year prior beliefs about θ are described by a distribution with density $f(\theta) \alpha \; \theta^{\alpha-1} (1-\theta)^{\alpha-1}$ for some constant $\alpha > 0$.

(a) (i) Derive the maximum likelihood estimate $\hat{\theta}$ of θ given the data **8**

$x_1, x_2, ... x_n$.

(ii) Derive the posterior distribution of θ given $x_1, x_2, ... x_n$.

(iii) Derive the Bayesian estimate of θ under quadratic loss and show that it takes the form of a credibility estimate

$Z\hat{\theta} + (1-z)\mu$

where μ is a quantity you should specify from the prior distribution of θ.

(iv) Explain what happens to Z as the number of years of observed data increases.

(b) Determine the variance of the prior distribution of θ. **3**

(c) Calculate the Bayesian estimate of θ under quadratic loss if $n=3$ $x_1=3$, $x_2=3$ and $x_3=5$ and **4**

(i) $\alpha=5$

(ii) $\alpha=2$

Comment on your result in the light of (b) above.
