M.Sc. ACTUARIAL SCIENCE

Term-End Examination
December, 2011

MIA-009 (F2F) : GENERAL INSURANCE, LIFE AND HEALTH CONTINGENCIES

Time : 3 hours   *Maximum Marks : 100*

Note : *In addition to this paper you should have available Actuarial Table and your own electronic calculator.*

·SECTION - A

(Answer *any five* questions)

1. By considering a term assurance policy as a series   8
    of one year deferred term assurance policies, show
    that

    $$\overline{A}^1_{x\, :\, \overline{n}|} = \frac{i}{\delta}\, A^1_{x\, :\, \overline{n}|}$$

2. Prove Thieles differential equation for a whole life   8
    assurance issued to a life aged x to be as follows

    $$\frac{\partial}{\partial t}\left[{}_t\overline{V}_x\right] = -\left(1 - {}_t\overline{V}_x\right)m_{x+t} + \delta\ {}_t\overline{V}_x + \overline{P}_x$$

3. (a) Describe the four different methods of **6** allocating bonuses to with profit contracts.

   (b) Explain how they differ in terms of rate of **2** distribution of surplus.

4. A unit - linked policy has the following profit vector

Year	In force profit
1	-25
2	-12
3	-6
4	25
5	35

   (a) Calculate the reserves required to zeroise **2** losses at the end of years 2 and 3, assuming a rate of accumulation of 8% and that $q_x = 0.01$ at each age.

   (b) If the risk discount rate used is 10% **6** determine the net present value of the profits before and after zeroisation and state with reason which of these figures you would expect to be greater.

5. For a certain group of pensioners, $q_{75} = 0.05$ and $q_{76} = 0.06$
   Calculate the probability that a pensioner aged 75 exact will die between ages 75.5 and 76.5 assuming :

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(a) a uniform distribution of deaths between 8 consecutive birthdays.

(b) a constant force of mortality between consecutive birthdays.

6. A certain population is subject to three modes of decrement : α, β, γ.

(a) Write down an expression for $(aq)_x^\alpha$ in terms 2 of the single decrement table probabilities q_x^α, q_x^β and q_x^γ, assuming each of the three modes of decrement is uniformly distributed over the year of age x to $x+1$ in the corresponding single decrement table.

(b) Suppose now that on the single decrement 6 table α, $tp_x^\alpha = 1 - t^2 q_x^\alpha (0 \le t \le 1)$ while decrements β and γ remains uniformly distributed. Drive a revised expression for $(aq)_x^\alpha$ in terms of the single decrement table probabilities q_x^α, q_x^β and q_x^γ.

7. (a) Define the following terms : 3
    (i) death strain at risk
    (ii) expected death strain
    (iii) actual death strain

(b) On 1 January 2005 a pension scheme has 5 100 members aged 75 exact, each eligible for a pension of Rs 10,000 p.a. payable annually in advance. In addition, the members were entitled to a death benefit of Rs. 20,000 payable at the end of the year of death. No premiums were being paid in respect of these contracts after January 2005. Given that 4 of the lives died during 2005, calculate the mortality profit for these contracts for calendar year 2005 using the following basis :

Mortality : PFA 92 C 20

Interest : 4% p.a

Expenses : none.

SECTION - B

(Answer any **four** questions)

8. Let K denote the curtate future lifetime random variable of a life aged exactly x.

    (a) Describe the benefit whose present value random variable is   **2**

    $$W = \begin{cases} 10000\, \ddot{a}\, \overline{k+1} & \text{if } k > 10 \\ 10000\, \ddot{a}\, \overline{10} & \text{if } k \geq 10 \end{cases}$$

    (b) Prove the premium conversion formula   **3**

    $$A x : \overline{n} = 1 - d\, \ddot{a} x : \overline{n}$$

    (c) Calculate the expected present value and the standard deviation of the present value of the benefit in (i), assuming   **10**

    *   a constant force of 0.04 p.a

    *   the life is subject to a constant force of mortality of 0.02 p.a

9. A life insurance company issues a 4 - year unit - linked endowment policy to a life aged 50 exact   **15**

under which level premiums of Rs. 750 are payable yearly in advance throughout the term of the policy or until earlier death. In the first policy year, 25% of the premium is allocated to units and 102.5% in the second and subsequent years. The units are subject to a bid offer spread of 5% and an annual management charges of 1% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

If the policy holder dies during the term of the policy, the death benefit of Rs 3000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender the bid value of the units is payable at the end of the policy year of exit. On maturity, 110% of the bid value of the unit is payble.

The company uses the following assumptions in carrying out profit tests of this contract :

Rate of growth on assets in the unit fund	:	6.5% p.a
Rate of interest on non-unit fund cash flows	:	5.5% p.a
Mortality	:	AM 92 select
Initial expenses	:	Rs 150
Renewal expenses	:	Rs 65 p.a on the second and subsequent premium dates.
Initial commission	:	10% of the first premium
Renewal commission	:	2.5% of the second and subsequent years premiums.
Risk discount rate	:	8.5% p.a

In addition assume that at the end of each of first 3 years, 10% of all policies still in force then surrender.

Calculate the profit margin for the policy on the assumption that the company does not zeroise future expected negative cash flows.

10. A life office issues with-profit whole of life   **15**
contracts, with the sum assured payable
immediately on death of the life assured. Level
premiums are payable monthly in advance to age
65 or until earlier death.

The life office markets two version of this policy, one assumed to provide simple bonuses of 4% per annum of the sum assured vesting at the end of each policy year, and the other assume to provide compound bonuses of 4% of the sum assured, again vesting at the end of each policy year. The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is assumed to price these contracts :

Mortality :	AM 92 select
Interest :	4% per annum
Initial expenses :	Rs. 300
Renewal expenses :	2.5% of the second and subsequent monthly premiums
Initial commission :	50% of the gross annual premium
Renewal commission :	2.5% of the second and subsequent monthly premiums
claim expenses :	Rs. 250 at termination of the contract.

Calculate the level monthly premium required for each version of this policy issued to a life aged 30 exact at outset for an initial sum assured of Rs. 50,000.

11. A life insurance company issues an annuity 15
    contract to a man aged 65 exact and his wife aged
    62 exact.

    Under the contract, an annuity of Rs 20,000 p.a
    is guaranteed payable for a period of 5 years and
    there after during the life time of the man. On
    the man's death, an annuity of Rs 10,000 p.a is
    payable to his wife, if she is then alive. This
    annuity commences on the monthly payment
    date next following, or coincident with, the date
    of his death or from the 5th policy anniversary, if
    later and is payable for the lifetime of his wife.
    Annuities are payable monthly in advance.

    Calculate the single premium required for the
    contract basis :

    Mortality : PMA 92 C 20 for the male and

                  PFA 92 C 20 for the female

    Interest   : 4% p.a

    Expenses   : none

12. A man aged exactly 40, is a member of a pension 15
    scheme that pays a retirement pension of $\frac{1}{60}$th of
    final pensionable salary for each past year of

service. Final pensionable salary is defined to be the average salary earned over the previous three years. Retirement can take place at any age between 60 and 65. The member has earned Rs. 20,000 over the past year, and currently has 5 years past service. Defining all your terms, and starting any assumptions made, derive expressions in terms of suitable commutation functions for valuing the past service benefit and future service benefit for this member.

13. (a) Discuss how time selection and class selection can affect the result of a mortality investigation and how you consider these factors should be allowed for in developing mortality tables for practical use. **15**

(b) You are given the following statistics in respect of the population of Urbania :

Males

Age band	Exposed to risk	observed mortality rate
20 - 29	1,25,000	0.00356
30 - 39	2,00,000	0.00689
40 - 49	1,00,000	0.00989
50 - 59	90,000	0.01233

Females

Age band	Exposed to risk	observed mortality rate
20 - 29	1,00,000	0.00125
30 - 39	2,50,000	0.00265
40 - 49	2,00,000	0.00465
50 - 59	1,50,000	0.00685

Calculate the directly and indirectly standardised mortality rates for the female lives using the combined population as the standard population.

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