

**M.Sc. ACTUARIAL SCIENCE
(MSCAS)**

Term-End Examination

December, 2011

MIA-001 (F2F) : FINANCIAL MATHEMATICS

Time : 3 hours

Maximum Marks : 100

Note : *In addition to this paper you should have available the Formula and Table and your own electronic calculator.*

SECTION - A

(Answer *any five* questions)

1. (a) Given $i = 7.5\%$ p.a, find
- (i) $(\ddot{I}a)_{\overline{15}|}$ 2
- (ii) $(Is)_{\overline{10}|}$ 2
- (b) Explain when you should use real rates and money rates of interest. Give an example of when each rate of interest would be used. 4
2. (a) A continuous payment stream is such that the level rate of payment in year t is $100 \times 1.05^{t-1}$, $t = 1, 2, \dots, 10$. Calculate the present value of the payment stream as at its commencement date, assuming a rate of interest of 10% p.a. 3

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- (b) Calculate the present value of an annuity payable annually in advance for a term of 20 years such that the payment is Rs. 500 in year 1, Rs. 550 in year 2, Rs. 600 in year 3 etc. Assume a rate of interest of 5% p.a for the first twelve years and 7% p.a there after. 5

3. The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years is given by

$$\delta(t) = \begin{cases} 0.07 - 0.005t & \text{for } t \leq 8 \\ 0.06 & \text{for } t > 8 \end{cases}$$

- (a) Calculate the accumulation at time $t=10$ of Rs. 550 invested at time $t=0$. 3
- (b) Calculate the present value at time $t=0$ of a continuous payment stream at the rate of Rs. 200 $e^{0.1t}$ paid from $t=10$ to $t=18$. 5

4. A computer manufacturer is to develop a new chip to be produce from 1 January 2008 until 31 December 2020. Development begins on 1 January 2006. The cost of development comprises Rs. 9 million payable on 1 January 2006 and Rs. 12 million payable continuously during 2007. From 1 January 2008 the chip will be ready for production and it is assumed that income will be received half yearly in arrear at a rate of Rs. 5 million pa.

- (a) Calculate the Discounted payback period at an effective rate of interest of 9% p.a. 6
- (b) Without doing further calculations, explain whether discounted payback period would be greater than, less than or equal to that given in part (i) if the effective interest rate were substantially greater than 9% p.a. 2
5. (a) Define interest rate swaps and describe briefly the two kind of risk facing each counterparty to the swap. 4
- (b) Give an outline of how corporate debt differs from government debt. 4
6. An investor entered into a long forward contract for a security five years ago and the contract is due to mature in seven years time. The price of the security was Rs. 95 five years ago and is now Rs. 145. The risk - free rate of interest can be assumed to be 3% per annum throughout the 12-year period.
- Assuming no arbitrage, calculate the value of the contract now if.
- (a) The security will pay dividends of Rs. 5 in two years' time and Rs. 6 in four years time. 4
- (b) The security has paid and will continue to pay annually in arrear a dividend of 2% per annum of the market price of the security at the time of payment. 4

7. In a particular bond market, n -year spot rates per annum can be approximated by the function $0.08 - 0.04 e^{-0.1n}$.

Calculate :

- (a) The price per unit nominal of a zero coupon bond with term nine years. 2
- (b) The four-year forward rate at time 7 years. 3
- (c) The three-year per yield. 3

SECTION - B

(Answer *any four* questions)

8. (a) Show that 5

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

- (b) An annuity certain payable continuously for a term of 10 years provides payments at the rate of 10

- * 1 p.a for the first 3 months
- * 2 p.a for the next 3 months
- * 3 p.a for the next 3 months, and so on.

To what amount will the annuity payments accumulate by the end of the term if interest is 10% p.a convertible half yearly.

9. (a) A loan is repayable over 20 years by level instalments of Rs. 1,000 p.a made annually in arrear. Interest is charged at the rate of 5% p.a effective for the first 10 years, increasing to 7% p.a effective for the remaining term. Show that the amount of the original loan is Rs. 12033.56. 3

- (b) The following are the details from the loan schedule for year x , i.e. the year running from exact duration $x-1$ to exact duration x years. 4

	Loan outstanding at the beginning of the year	Instalment paid at the end of the year Interest	Capital
Year x	Rs 8790.48	Rs 439.52	Rs 560.48

Determine the value of x .

- (c) At the beginning of year 11, it is agreed that the increase in the rate of interest will not take place, so that the rate remains at 5% p.a effective for the remainder of the loan. 8
- The annual instalment will continue to be payable at the same level so that there may be a reduced term and a reduced final instalment.
- (i) Calculate by how many years, if any, the payment schedule is shortened.
 - (ii) Calculate the amount of the reduced final instalment.
 - (iii) Calculate the reduction in the total interest paid during the existence of the loan as a result of the interest rate not increasing.

10. On 1 September 2001, a company placed part of its assets with two fund managers. Manager P was given Rs. 80,000 and manager Q was given Rs. 1,40,000. Both managers received a net cashflow of Rs. 15,000 on 1 September 2002, bringing their total fund values to Rs. 1,03,000 and Rs. 1,83,000 respectively.

A further net cash flow of Rs. 20,000 was received by each manager on 1 September 2003. This brought their total fund values to Rs. 1,43,000 and Rs. 2,39,600 respectively.

On 31 August 2004, the value of manager P's fund was Rs. 1,72,320 and the value of manager Q's fund was Rs. 2,63,560.

- (a) For the period from 1 September 2001 to 31 August 2004, calculate for each fund manager : 10
- (i) the time weighted rate of return
 - (ii) the money weighted rate of return
- (b) By examining the growth factors between cash flows, describe the performance of each manager over three-year period. Hence , explain why the money weighted rate of return for manager P is higher than that of manager Q ? 3
- (c) Comment briefly on the relative performance of the two fund managers. 2

11. A loan is issued bearing interest at a rate of 9% per annum and payable half-yearly in arrear. The loan is to be redeemed at Rs. 110 per Rs. 100 nominal in 13 years' time.

(a) The loan is issued at a price such that an investor, subject to income tax at 25% and capital gain tax at 30%, would obtain a net redemption yield of 6% per annum effective. Calculate the issue price per Rs 100 nominal of the stock. 5

(b) Two years after the date of issue, immediately after a coupon payment has been made, the investor decided to sell the stock and find a potential buyer, who is subject to income tax at 10% and capital gain tax at 35%. The potential buyer is prepared to buy the stock provided she will obtain a net redemption yield of at least 8% per annum effective. 10

(i) Calculate the maximum price (per Rs. 100 nominal) which the original investor can expect to obtain from the potential buyer.

- (ii) Calculate the net effective annual redemption yield (to the nearest 1% per annum effective) that will be obtained by the original investor if the loan is sold to the buyer at the price determined in (b) (i).

12. An insurance company has liabilities consisting of eleven annual payments of Rs. 1 million, with the first payment due to be made in 10 years' time and the last payment due to be made in 20 years time. The rate of interest is 6% per annum effective.

- (a) Show that the discount mean term of these liabilities, to four significant figures, is 14.42 years. 5

The insurance company holds two zero-coupon bonds, one paying Rs. X in 10 years time and the other paying Rs. Y in 20 years' time.

- (b) Find the value of X and Y such that Redington's first two condition for immunisation from small changes in the rate of interest is satisfied. 8

- (c) Explain, without making any further calculations, whether you would expect Redington's third condition for immunisation to be satisfied for the value of X and Y calculated in (b). 2

13. Rs. 80,000 is invested in a bank account which pays interest at the end of each year. Interest is always reinvested in the account. The rate of interest is determined at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year.

During the first year, the annual effective rate of interest will be one of 4%, 6% or 8% with equal probability.

During the second year, the annual effective rate of interest will be either 7% with probability 0.75 or 5% with probability 0.25.

During the third year, the annual effective rate of interest will be either 6% with probability 0.7 or 4% with probability 0.3.

- (a) Derive the expected accumulated amount on the bank account at the end of three years. 5

- (b) Derive the variance of the accumulated amount in the bank account at the end of three years. 7
- (c) Calculate the probability that the accumulated amount in the bank account is more than Rs. 97,000 at the end of three years. 3