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MMTE-005

00505

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination

December, 2011

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Do **any five** questions from question **1** to **6**. Use of Calculators are **not** allowed.*

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1. (a) Find the minimum distance and weight enumerator of the code generated by **6**

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Find its dual and find the minimum distance and weight enumerator of the dual code e^1.

(b) Let $f(x) = x^3 + x^2 + 1$ be an irreducible polynomial over F_2. Find all elements of **4**

$$F_8 = \frac{F_2[x]}{(f(x))}.$$

2. (a) Let e be a code with parity check matrix **6**

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Find the parity check matrix of the extended code. Also check that the extended code is a [8, 4, 4] code.

(b) Check that the code e with generator matrix **4**

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

is a perfect code.

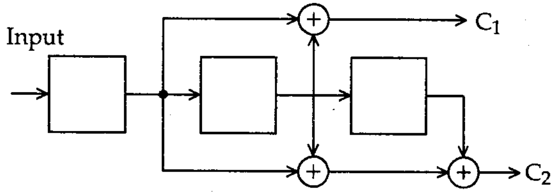
3. (a) If e is a self dual $\mathbf{Z}_4$ – linear code, prove that Res(e) is a doubly-even and $\text{Res(e)} = \text{T or(e)}^{1}$. **4**

(b) (i) Define cyclic code and give an example. **6**

(ii) Let $g(x) = 1 + x + x^3$ be the generator polynomial of a [7, 4] cyclic code. Write its parity check matrix and generator matrix.

4. (a) Find [15, 7, 5] and [15, 5, 7] BCH codes over F2. (You have to find the generator matrix of the codes and verify that the parameters are as specified.) 6

(b) Find the convolutional code for the message 1001. The convolutional encoder is given below : 4



5. (a) Prove that, for any [n, k] code e and its dual, the weight distributions satisfy the equation 4

$$\sum_{j=0}^{n-2} \binom{n-j}{\nu} A_j = q^{k-\nu} \sum_{j=0}^{\nu} \binom{n-j}{n-\nu} A_j^1$$

for $0 \leq \nu \leq n$.

(b) Explain the soft decision viterbi algorithm. 6

6. (a) If duadic codes of length n over $\mathbf{F}_q$ exists, prove that q is a square modulo n. 5

(b) Write down the generator matrix of a [7, 4] Hamming code. How many errors can it detect and how many errors it correct. 5