

**M.Sc. (MATHEMATICS WITH APPLICATIONS
IN COMPUTER SCIENCE) (MACS)
M.Sc. (MACS)**

Term-End Examination

December, 2011

**MMTE-002 : DESIGN AND ANALYSIS OF
ALGORITHMS**

Time : 2 hours

Maximum Marks : 50

Note : Answer any five questions. Calculators are not allowed.

1. (a) Does the array 5

3	6	2	5	2	8	2	1	4	1
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represent a max heap? Justify your answer.

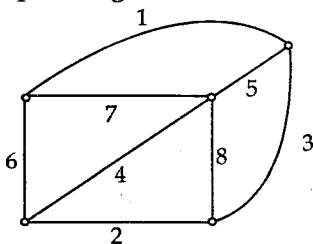
If it doesn't, show how this can be converted into a max heap using MAX-HEAPIFY algorithm. Represent the max heap as a binary tree.

- (b) Let $T(n)$ be a function satisfying the recurrence 3

$$T(n) = T\left(\left\lceil \frac{n}{2} \right\rceil\right) + T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1, T(1) = 1.$$

Prove by induction that $T(n)$ satisfies a bound of the form $T(n) \leq (n - b)$ for appropriately chosen values of c and b . Also find appropriate values of b and c .

- (c) For carrying out certain task, there are two algorithms, A and B. Algorithm A takes $n^2 + 1000$ operations to complete the task and algorithm B takes $65n$ operations to complete the task, where n is the size of the input. Find a no $\epsilon \mathbb{N}$ such that for any input of size greater than n_0 , algorithm B performs better than algorithm A. 2
2. (a) Find an optimal parenthesisation of a matrix chain product whose sequence of dimensions is given by $\{4, 6, 30, 8, 9\}$. 5
- (b) Illustrate all steps of Robin - Karp - Miller string matching algorithm for $P = 1035$, $T = 140610216$. and $Q = 7$. 5
3. (a) Explain the algorithm design divide and conquer approach using merge sort. 2
- (b) Illustrate MERGE procedure of MERGE SORT using the array $\{1, 4, 6, 7\}$ and $\{2, 5, 8, 9, 11\}$. 3
- (c) Explain the Prim's algorithm using pseudo code for finding the minimal spanning tree in a graph. Use it to find the minimal spanning tree in the following graph. 5



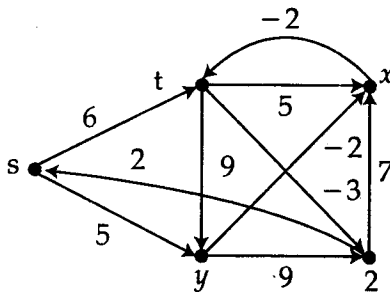
4. (a) Illustrate the procedure PARTITION, used in quick sort, using the array. 4

$$A = \{3, 5, 2, 7, 1, 4\}$$

- (b) Explain with the help of an example that greedy strategy does not always yield optimal solution for optimisation problems. 4

- (c) Compute the DFT of the vector $(2, 2, -1, 1)$ 2

5. (a) Explain why there is no solution for the shortest path problem in a weighted directed graph if it contains a cycle of negative weight. Explain the Bellman - Ford algorithm with the help of the following graph with r as the source vertex. 6



- (b) Explain the counting sort algorithm using the array 4

$$A = \{1, 3, 2, 1, 5, 2, 6\}$$

6. (a) Explain the INORDER - TREE - WALK (x) procedure with pseudo code. Show that, if x is the roof of an n - node sub tree, then the call INORDER - TREE - WALK (x) takes $(H) (n)$ times. 6
- (b) Find $7^{562} \pmod{563}$ using modular exponentiation algorithm. 4
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