No. of Printed Pages : **4**

MMTE-001

00535

M.Sc. (Mathematics with Applications in Computer Science) (MACS)

Term-End Examination

December, 2011

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. 1 is **compulsory** and answer **any four** from the rest six (2 - 7). Calculators and similar devices are **not** allowed.*

1. State, giving justifications or illustrations, whether each of the following statements is *true* or *false*. **5x2=10**

(a) The isomorphism relation on the set of all simple graphs is an equivalence relation.

(b) K_n is not bipartite for $n \geq 3$.

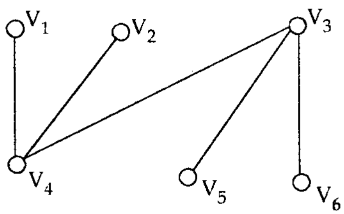
(c) Every four colourable graph is planar.

(d) There exists a tree with degree sequence (3, 3, 2, 2, 2).

(e) Every Hamiltonian graph is Eulerian.

2. (a) Draw a regular simple graph G with 9 vertices and 18 edges. **4**

(b) In the graph given below give the following with justification. **6**



(i) A matching of maximum size.

(ii) A vertex cover of minimum size.

(iii) An independent set of vertices of maximum size.

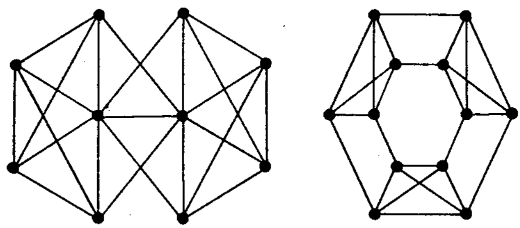
3. (a) Use induction (on n) to prove that if $d_1, d_2, \ldots\ldots d_n$ are non - negative integers and Σd_i is even, then there is an n-vertex graph with vertex degrees $d_1, d_2, \ldots, d_n$. **5**

(b) Explain how to construct the Priifer code of a tree ? Show that distinct trees have distinct Priifer codes. **5**

4. (a) Define the Hamiltonian closure of a graph. Show that if the closure of a graph G is Hamiltonian, then G is Hamiltonian. 5

(b) Show that if G is Eulerian, its line graph is Hamiltonian. Give a counter example to show that the converse is not true. 5

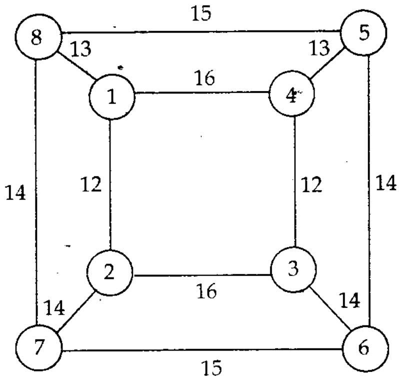
5. (a) Determine $k(G)$, $k^1(G)$ and $\delta(G)$ for each of the graph given below : 5



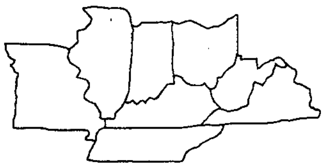
(b) Prove that a planar graph G is bipartite if and only if every face G has even length. 5

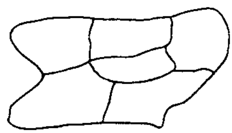
6. (a) Prove that a graph G in k-partite if and only if it is k-colourable. 4

(b) Describe Prim's algorithm to find a minimum spanning tree. Apply Prim's algorithm to find a minimum spanning tree for the graph given below : **6**



7. (a) Consider the problem of colouring the regions of the following maps : **4**





For each of the maps draw the corresponding graph and find its chromatic number.

(b) Draw the graph whose incidence matrix is **6**

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

What is the adjacency matrix of this graph.