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**MMT-009**

00305

# M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

## Term-End Examination

## December, 2011

## MMT-009 : MATHEMATICAL MODELLING

*Time : 1½ hours* *Maximum Marks : 25*

***Note :*** *Do **any five** questions. Use of calculator is **not** allowed.*

**1.** (a) Assume that the return distribution of two securities X and Y be as given below : **3**

| Possible rates of returns of security | | Associated Probabilities |
|---|---|---|
| X | Y | $p_{xj}=p_{yj}$ |
| 0.11 | 0.18 | 0.42 |
| 0.17 | 0.16 | 0.15 |
| 0.10 | 0.11 | 0.30 |
| 0.19 | 0.09 | 0.13 |

Find $\rho_{XY}$.

(b) In a population of animals, the Proportionate birth rate and death rate are both constant, being 0.15 per year and 0.50 per year respectively. Formulate a model of population and discuss its long term behaviour. **2**

2. (a) Find a linear demand curve that best fit the following data : 3

| X | 20 | 22 | 24 | 26 | 28 | 30 | 32 |
|---|---|---|---|---|---|---|---|
| Y | 50 | 55 | 40 | 35 | 30 | 60 | 25 |

(b) Four securities have the following expected returns : 2

A = 17%, B = 14%, C = 25%, D = 22%.

Calculate the expected return for a portfolio consisting of all four securities, where the portfolio weights are 40% each in A and B and 10% each in C and D.

3. (a) The reproduction function of the cancer cell within a spherical tumour is given by 3

$$\phi(C) = \frac{C-1}{2(1-2C)^2} \; ; C \neq \frac{1}{2}$$

with initial condition given by C = 3 at t = 0. Find the density of the cancer cells in the tumour's surface area when t = 60 days.

(b) A simple model including the seasonal change that affects the growth rate of a population is given by $\frac{dx}{dt} = C\, x(t)$ cost, where C is a constant. If $x_0$ is the initial population then solve the equation and determine the maximum and minimum populations. 2

4. The population dynamics of a species is governed by the discrete model 5

$$x_{n+1} = x_n \exp\left[r\left(1 - \frac{x_n}{k}\right)\right]$$

where r and k are positive constants. Determine the steady-states and discuss the stability of the model. Find the value of r at which first bifurcation occurs.

5. Consider the following prey and predator interacting system under the effect of toxicant, where the concentration of the toxicant in the environment is assumed to be constant. 5

$$\frac{dN_1}{dt} = r_0 N_1 - r_1 C_0 N_1 - bN_1 N_2$$

$$\frac{dN_2}{dt} = -d_0 N_2 + \beta(C_0) bN_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

along with the initial conditions :

$N_1(0)=N_{10}$, $N_2(0)=N_{20}$, $C_0(0)=0$ and

$\beta(C_0)=\beta_0 - \beta_1 C_0$, where $\beta(C_0)$ is the conversion coefficient depending upon $C_0$. The variable and parameters notations in the above system of equations are :

$N_1(t)$=Density of prey population

$N_2(t)$=Density of predator population

$C_0(t)$=Concentration of toxicant in the individuals of the population.

P=Concentration of toxicant in the environment and is constant.

$r_0$ is the birth rate, $r_1$ is the death rate due to $C_0$, b is the predation rate, $d_0$ is the death rate, $k_1$ is the up take rate, $g_1$ is the loss rate of the toxicant and $m_1$ is the depuration rate. Here $r_0$, $r_1$, b, $d_0$, $k_1$, $g_1$ and $m_1$ are all positive constants. Reformulate the above system under the assumption that the predators are not affected by the toxicant because they are generally strong and then do the stability analysis of the reformulated model.

6. (a) Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of inter-arrival times. The time a ship occupies a birth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for berths is to be kept below 14 hours, how many berths should be provided at the port ? 3

(b) Explain each of the following with examples : 2

(i) Hurvitz criteria.

(ii) Multiple linear regression model with k predictors.

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