MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination 00639

December, 2011

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. 1 is **compulsory**. Do any **four** questions out of the remaining questions 2-7. All computations may be kept to 3 decimal places. Use of calculators is **not** allowed.*

State whether the following statements are **true** or **false**. Justify your answer with the help of a short proof or a counter example. **2x5=10**

1. (a) For the differential equation

$$x^2 (x-2)^2 y'' + 3 (x-2) y' + (x+7) y = 0,$$

$x=0$, is an irregular singular point.

(b) If $\int_{-1}^{1} P_n (x)\, dx = 2$, then n is 1.

(c) The inverse Fourier transform of

$$\frac{1}{(i\alpha + k)^2}, k > 0$$

is $\int_{-\infty}^{\infty} e^{-k\tau} H(\tau) e^{-k(x-\tau)} H(x-\tau)\, d\tau.$

where H(τ) is the Heavi side step function.

(d) The interval of absolute stability of the Euler's method is $-2 < \lambda h < 0$.

(e) The function $y(t)$ satisfying the integral equation

$$y(t) = \sin 2t + \int_0^t y(t) \sin 2(t-\tau)\, d\tau \text{ is}$$

$$y(t) = \sqrt{2} \sin \sqrt{2}\, t.$$

2. (a) Using Laplace transform technique solve the following initial value problem 5

$$y_1'' = y_1 + 3y_2 \; ; \; y_2'' = 4y_1 - 4e^t$$

$$y_1(0) = 2,\; y_1'(0) = 3 \; ; \; y_2(0) = 1,\; y'_2(0) = 2.$$

(b) Prove that 5

$$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

3. (a) Solve the following differential equation by the power series method about $x=0$: **5**

$$x^2y'' + (x^2 + x)y' + (x - 9)\ y = 0.$$

(b) If $f(x) = 3e^{-|2x|} - 4e^{2|x+3|}$, find the Fourier transform of $f(x)$. **2**

(c) Using inverse Fourier cosine transform, find $f(x)$ if **3**

$$F_c(\alpha) = \begin{cases} \left(a - \frac{\alpha}{2}\right), & \alpha \le 2a \\ 0, & \alpha > 2a \end{cases}.$$

4. (a) Expand $f(x) = x^2$ in terms of Fourier-Bessel series in the interval $0 < x < 2$, in terms of $J_2(\alpha_n x)$ where α_n are the roots of $J_2(2\alpha) = 0$. **5**

(b) Derive the method **5**

$y_{i+1} = a_1 y_i + a_2 y_{i-1} + hb_0 y'_{i+1}$ for solving the initial value problem

$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. Find the truncation error and the order of the method.

5. (a) Using Crank - Nicholson's method **7**

solve $u_t = \frac{1}{16} u_{xx}$, $0 < x < 1$, $t > 0$

given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 50t$

Take $\lambda = \frac{1}{16}$ and $h = \frac{1}{4}$.

(b) Find the solution of equation **3**

$y'' + \frac{1}{x} y' + \left(1 - \frac{1}{4x^2}\right) y = 0$ in terms of sine and cosine functions.

6. (a) Construct Green's function for the following boundary value problem : **5**

$$\frac{d^2 y}{dx^2} - k^2 y = e^{3x}, \; y(0) = y(1) = 0.$$

(b) Using second order finite difference method with $h = \frac{1}{2}$, obtain the system of equations for y_0, y_1, y_2 for solving the boundary-value problem : **5**

$$y'' - 5y' + 6y = 3,$$

$$y(0) - y'(0) = -1, \; y(1) + y'(1) = 1.$$

7. (a) Find $y(0.1)$, $y(0.2)$, $y(0.3)$ from **5**

$$\frac{dy}{dx} = x^2 - y, \; y(0) = 1$$

by using Taylor series method. Hence obtain $y(0.4)$ using Adam-Bashforth method with predictor P and corrector C as follows :

$$P : y^{P}_{n+1} = y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

$$C : y^{C}_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} - 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$

(b) Consider a steel plate of size 15 cm × 15 cm. If two of the sides are held at 100°C and the other two sides are held at 0°C, determine the steady state temperature at interior points assuming a grid size of 5 cm × 5 cm. Use five - point formula. **5**
