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MMT-006

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2011

MMT-006 : FUNCTIONAL ANALYSIS

Time : 2 hours *Maximum Marks : 50*

Note : *Answer question number **1** which is **compulsory**. Attempt **any four** of the remaining six questions.*

1. Are the following statements ***true*** or ***false*** ? Justify your answer with the help of a short proof or counter example. **5x2=10**

(a) The sum of two norms on a vector space is again a norm.

(b) The space of all polynomials with real coefficients can be made into a Banach space with a suitable norm.

(c) The dual space of l^{∞} is isometrically isomorphic to l^1.

(d) If $\{x_1, x_2, \ldots\ldots, x_n\}$ is an orthogonal set in a Hilbert space H, then $\|\Sigma x_i\|^2 = \sum_{i=1}^{n} \| x_i \|^2$.

(e) If A and B are positive operators on a Hilbert space H, then so is AB.

2. (a) State Hatin Banach Extension Theorem. Give an example to show that Hatin Banach extensions need not be unique. 4

(b) If every absolutely convergent series in a normed linear space X is convergent, then show that X is complete. 3

(c) If A is a bounded linear operator A on a Hilbert space H satisfies the condition that $\|Ax\| = \|A^*x\|$ for all x in H, prove that A is normal. 3

3. (a) Show that the map f defined on L'[0, 1] by $f(x) = \int_0^1 t\ x(t)dt$, for x in L'[0, 1] is a bounded linear functional on L'[0, 1]. 3

(b) Show that a subspace of a reflexive space is reflexive if and only it is closed. 4

(c) Show that a subspace S of a Hilbert space H is dense if and only if $S^{\perp} = (0)$. 3

4. (a) Let X_1 be a Banach space, X_2 a normed linear space and τ a subset of BL(X, Y) such that for each $x \in X_1$, the set $\{T(x) : T \in \tau\}$ is bounded in X_2. Then for each bounded subset S of τ, show that the set $\{T(x) : x \in S, T \in \tau\}$ is bounded in X_2. 5

(b) Let $M = \{x \in l^2 : x(2n) = 0 \text{ for all } n\}$. What is $M^{\perp}$? 3

(c) Let X be a finite dimensional space. Then show that any operator $T : X \rightarrow X$ is compact. 2

5. (a) Give two inequivalent norms on C'[0, 1]. Justify your answer. 4

(b) Let X be a Banach space and Y be a closed subspace of X. Show that X/Y is a Banach space. 3

(c) If M is a subspace of a Hilbert space H, show that $M^{\perp} = M^{\perp\perp\perp}$. 3

6. (a) Find a bounded linear map $T : C[0, 1] \rightarrow C[0, 1]$ with $\sigma(T) = [0, 1]$. 3

(b) Let X be an inner product space and let $x, y \in X$. Suppose that $\{a_n\}$ is a sequence of scalars converging to a scalar a, prove that $\langle a_n x, y \rangle \rightarrow \langle a x, y \rangle$ in X. 2

(c) Identify the dual space of l'. 5

7. (a) Prove that any compact self adjoint operator on a Hilbert space has an eigen value. 3

(b) Prove or disprove : Any linear map between normed linear spaces with closed graph is bounded. 3

(c) Let $T : X \to Y$ be a linear transformation between the two normed spaces X and Y. If T is continuous at O, then show that T is uniformly continuous. Give an example to show that this result is not true if the linearity condition of T is dropped. 4