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MMT-005

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2011

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

Note : *Question **No. 1** is **compulsory** . Attempt **any three** questions from question number **2** to **5**. Use of calculator is **not** allowed.*

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1. State giving reasons whether the following statement are true or false : **5x2=10**

(a) If $f(z)=\dfrac{e^z+\sin z}{(Z+2)^{100}}$ then $\oint_C f(z)=\dfrac{\pi}{(4)^{100}}$

where C is the circle $|z|=\dfrac{\pi}{2}$.

(b) The radius of convergence of the complex power series $\sum\limits_{n=1}^{\infty}\left(\dfrac{6n+1}{2n+5}\right)^n (z-2i)^n$ is 3.

(c) For any two complex numbers z_1 and z_2 $\text{Arg}\,(z_1z_2)=\text{Arg}(z_1)+\text{Arg}(z_2)$ where Arg (z) denotes the principal argument of z.

(d) $f(z)=\dfrac{z}{e^z-1}$ has removable singularity at the origin $z=0$.

(e) The linear fractional transformation mapping points -1, 0, 2 onto points 0, 1, ∞ is $\dfrac{2z+2}{-z+2}$.

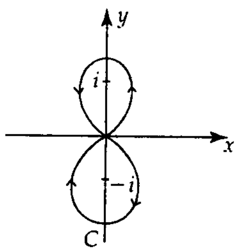
2. (a) Verify that the function 3

$u(x, y)=x^3-3xy^2-5y$ is harmonic in the entire complex plane and find the harmonic conjugate function of u.

(b) If $f(z)=-iz+i$ for $|z|\leq 5$ then show that $f(z)$ attains its maximum value when $z=-5$. 2

3. (a) Evaluate the given integral where C is the contour in the following figure : 3

$$\oint_C \frac{e^{iz}}{(z^2+1)^2}\,dz$$



(b) Expand $f(z)=\dfrac{\cos z}{z}$ in Laurent series valid for the region $|z|>0$. Write down the principal part of the series obtained. 2

4. (a) Find the poles of the function 2½

$$f(z) = \frac{1}{(z-1)^2(z-3)}$$ and residues at these poles. Hence evaluate $\oint_C f(z)\,dz$ where C is the circle $|z| = 2$.

(b) Define conformal mapping. Find all points where the mapping $f(z) = (z^2 + 1)\ e^z$ is conformal. 2½

5. Evaluate the Cauchy principal value of 5

$$\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - 2x + 2)}\,dx.$$
