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MMT-004

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2011

00859

MMT-004 : REAL ANALYSIS

Time : 2 hours *Maximum Marks : 50*

Note : *Question no. 1 is **compulsory**. Do **any four** questions out of questions no. 2 to 7.*

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1. State, whether the following statements are **TRUE** or **FALSE**. Give reasons for your answer. **5x2=10**

(a) If (x_1, d_1) and (x_2, d_2) are two discrete metric spaces, then the product metric on $X_1 \times X_2$ is discrete.

(b) The set [0, 1) U [1, 2] is a connected subset of R with respect to the standard metric.

(c) The function $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $f(x, y) = (x^2 - y^2, 2xy)$ is not invertable.

(d) The union of an open subset of **R** and a closed subset of **R** can not be measurable.

(e) The function $f : \mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \begin{cases} 1, \text{ when } x \in \left[n, n+\frac{1}{2}\right) \\ \quad n \in z \\ 0, \text{ when } x \in \left[n+\frac{1}{2}, n+1\right) \end{cases}$$

is continuous almost everywhere on **R**.

2. (a) Let d and D be metrices defined on $\mathbf{R} \times \mathbf{R}$ as **3**

$d\left((x_1, x_2), (y_1, y_2)\right) = |x_1 - y_1| + |x_2 - y_2|$

$D\left((x_1, x_2), (y_1, y_2)\right) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$

Show that d and D equivalent metrices on $\mathbf{R}^2$.

(b) State Dominated convergence Theorem. **4**

Use this theorem to find $\lim_{n \to \infty} \int_0^1 fn(x)\, dx$

where $fn(x) = \dfrac{x}{1 + n^2 x^2}$.

(c) Find the convolution f * g of the following functions f and g : **3**

$$f(t) = \frac{t^{1/3}}{1 - 2(1-t)^{1/3}}, \quad 0 < t < 1$$

$$g(t) = 1 - 2t^{1/3}, \quad 0 < t < 1$$

3\. (a) Let (x, d_1) and (y, d_2) be metric spaces. Let $x_0 \in x$. If $f(x, d_1) \mapsto (y, d_2)$ is continuous at x_0 then show that for every sequence $\{x_n\}$ in x converging to x_0, the sequence $\{f(x_n)\}$ converges to $f(x_0)$ in y. Is the converse true ? Justify your answer. 5

(b) Find the Fourier series for the function 3

$$f(t) = \begin{cases} -5, \text{ if } -\pi < t < 0 \\ 5, \text{ if } 0 < t < \pi \end{cases}.$$

(c) Let $P_1 : R^3 \to R$ be the projection function defined by $P_1(x, y) = x \; \forall (x, y) \in R^2$. Show that P_1 is continuous. 2

4\. (a) State inverse function theorem. Use this theorem to check whether the function $f : R^4 \to R^3$ defined by $f(x, y, z) = (x + y + z, 2 e^y \cos x, 2 e^y \sin x)$ is locally invertable at $(1, 1, 1)$. 5

(b) Let $\{ E_n \}$ be a sequence of pairwise disjoint measurable sets. Show that 5

$$m\left(\bigcup_{1=1}^{\infty} E_i\right) = \sum_{1=1}^{\infty} m(E_i)$$

5. (a) Define the following in the context of signals and systems and give one example for each. Justify your choice of example. **4**

   (i) Time invariant.

   (ii) Casual system.

(b) Let the function $f : \mathbf{R}^4 \rightarrow \mathbf{R}^3$ be defined by $f(x, y, z, w) = (x^2 y, y^2 z, z^2 x)$. Find f '(a) at a = (1, 2, − 1, − 2) **3**

(c) Let (x, d) be a metric space. Let F be a closed subset and k be a compact subset of x. Show that $F \cap K$ is compact in x. **3**

6. (a) Let (x, d) be a metric space and A be a non-empty subset of x. Show that **4**

$$\overline{A} = \{x : d(x, A) = 0\}.$$

(b) For the following sets, find m*A and check whether they are measurable. **3**

   (i) $A = \{0, 1\} \cup \{ x : x$ is a solution of $\sin x = 0\}$

   (ii) $B = \{ x : x$ is rational in $[2, 3] \}$

(c) Let the frequency response H (i W) be given **3**

by $H(i W) = \begin{cases} 1 & -W < w < W \\ 0 & \text{otherwise} \end{cases}$

Find the system response to the signal

$$h(t) = e^{i\frac{\omega}{7}t} + 4e^{i\frac{\omega}{5}t} + 7e^{i2\omega t}$$

7. (a) Show that the continuous image of a compact connected metric space is both compact and connected. 6

(b) Find the extreme values of the function $f(x, y, z) = xyz$ subject to the constraint 4

$$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} = 1, \; x, y, z \geq 0.$$