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MMT-003

00415

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2011

MMT-003 : (ALGEBRA)

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. 1 is **compulsary**. Also do **any four** questions from q. no. 2 to q. no. 6. Calculators are **not** allowed.*

1. Which of the following statements are **true** and which are **false**. Give reasons for your answer. **10**

   (a) There is a simple group of order 20.

   (b) It K/F is a field extension of degree 3, then $K = F(\alpha)$ for any $\alpha \in K - F$.

   (c) If χ is a character of a finite group G and $\bar{\chi}$ is defined by $\bar{\chi}(g) = \overline{\chi(g)}$, then $\bar{\chi}$ is also a character.

   (d) The field of order 64 has a sub field of order 32.

   (e) If ρ is a representation of a group G over **R**, then there is one and only one $g \in G$ for which $\rho(g) = I$.

2. (a) Let K C H C G be groups where G is a finite group. Prove the formula. 4

$$[G : K] = [G : H]\,[H : K]$$

(b) Solve the set of congruences 3

$x \equiv 1 \pmod 4$

$x \equiv 0 \pmod 3$

$x \equiv 5 \pmod 7$.

(c) Let G be a group of order n which operates non-trivially on a set of order r. If n+ r!, then show that G has a proper normal subgroup $\neq \{e\}$. 3

3. (a) Reduce the matrix given below to diagonal form by integer row and column operations : 3

$$\begin{bmatrix} 3 & 1 & -4 \\ 2 & -3 & 1 \\ -4 & 6 & -2 \end{bmatrix}$$

(b) Evaluate the Legendre symbol. 3

$$\left(\frac{13}{227}\right)$$

(c) Let $V = \{e, a, b, c\}$ be the klein four group with the group operation given below : 4

.	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Let $\rho : V \to GL_2(\mathbb{C})$ be defined as

$$\rho(e) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \rho(a) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho(b) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \rho(c) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(i) Check that ρ is a representation of V.

(ii) Find the character of ρ.

(iii) Is ρ irreducible ? Justify.

4. (a) Let $g \in An$ be an even permutation. Let C_1 and C_2, respectively denote the conjugancy classes of g in Sn and An. Further, let H_1 denote the centralizer of g in Sn. Show that if $|C_1| = |C_2|$, then H_1 contains odd permutations and if $|C_1| = 2|C_2|$ then H_1 does not contain any odd permutation. 4

(b) Check if the ISBN number 978 – 81 – 224 – 3129 – 2 is a valid ISBN number. 2

(c) Determine the irreducible polynomial for $\alpha = \sqrt{3} = \sqrt{5}$ over each of the following fields (i) $\mathbf{Q}$ (ii) $\mathbf{Q}(\sqrt{5})$. 2+2

5. (a) Let F be a field and H be a finite subgroup of the multiplicative group F^* of non-zero elements of F, Prove that H is a cyclic group. 6

(b) The matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ has order 3 and therefore, it defines a matrix representation $\{I, A, A^2\}$ of the cyclic group G of order 3. Find a G-invariant form an C^2. **4**

6. (a) Let α be a complex root of the irreducible polynomial $x^3 - 3x + 4$. Find the inverse of $\alpha^2 - \alpha + 1$ in **Q** (α) explicitly in the form $a + b\,\alpha + c\,\alpha^2$, a, b, c $\in$ **Q**. **3**

(b) Decide whether or not i is in the field $Q(\sqrt{-2})$. **2**

(c) Prove that the only one dimensional complex representations of S_5 are trivial representation defined by $\rho(g) = 1 \ \forall \ g \in S_5$ and the sign representation. **5**