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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2011

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 5 is* ***compulsory****. Answer* ***any three*** *questions from questions* ***1*** *to* ***4****. Use of calculators is* ***not allowed****.*

1. (a) Let T be a linear operator from R^3 to itself whose matrix with respect to the ordered basis **3**

$$\left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\} \text{ is } \begin{bmatrix}2 & 0 & 0\\0 & 5 & 0\\0 & 0 & 1\end{bmatrix}.$$

Find its matrix with respect to the ordered basis

$$\left\{ \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\}.$$

(b) The owner of a rapidly expanding business finds that for the first four months of the year, her sales are (in lakhs) Rs. 3.0, Rs. 6.3, Rs. 11.3 and Rs. 18 respectively. Find the least squares linear polynomial to fit the sales curve. **2**

2. (a) Find Let (e^A) if A is a 4×4 matrix with minimal polynomial $(t-3)^2$ and the rank of $(A-3I)$ is 2. **3**

(b) Check if $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ is unitarily diagonalisable. **2**

If it is find a unitary matrix V so that U* AV is a diagonal matrix otherwise find a unitary matrix V so that U*AV is upper triangular.

3. (a) Write the Jordan canonical form for **3**

$$A = \begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ Also, find the matrix P so

that $P^{-1}AP$ is the Jordan canonical form of A.

(b) Find the QR-decomposition of the matrix **2**

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

4. Find the Singular Value Decomposition of **5**

$$\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}.$$

5. Which of the following statements are true and which are false ? Give reasons for your answer. **2x5=10**

(a) If a non-singular matrix A is diagonalisable, A^{-1} is also diagonalisable.

(b) Two matrices with the same characteristic and minimal polynomials are similar.

(c) If N is a nilpotent matrix, then e^N is also nilpotent.

(d) Generalised inverse of an $m \times 1$ matrix need not be unique.

(e) Every diagonalisable matrix is normal.
