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01372

B.Tech. IN MECHANICAL ENGINEERING / B.Tech. IN CIVIL ENGINEERING

Term-End Examination

December, 2011

BICE-027 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

Note : *All the questions are to be answered in ENGLISH Language only. All the questions carry* ***equal*** *marks. Attempt* ***any SEVEN*** *questions.*

1. Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. **10**

Deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \text{-----} = \frac{\pi^2}{12}$$

2. Obtain the half - range sine series for the function $f(x) = x^2$ in the interval $0 < x < 3$. **10**

3. Find the Fourier transform of **10**

$$f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \leq 1 \\ 0, & \text{if } |x| > 1 \end{cases}$$

4. Find the Fourier cosine transform of the function $f(x)$. **10**

If $f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}$

5. Solve **10**

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x \partial y} - 2\frac{\partial^2}{\partial y^2}\right) z = (y-1)e^x.$$

Where symbols have their usual meaning.

6. Use the method of separation of variables to solve **10**

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

7. A string is stretched and fastened to two points **10**
'l' apart. Motion is started by displacing the string in the form $y = a \sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right).$$

Where $C^2 = \sqrt{\frac{T}{m}}$ i.e. one dimensional wave equation constant.

8. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. Which satisfies the conditions 10

$u(0, y) = u(l, y) = u(x, 0) = 0$ and

$$u(x, a) = \sin\left(\frac{n\pi x}{l}\right).$$

9. A rectangular plate bounded by the lines $x=0$, $y=0$, $x=a$, $y=b$ has an initial distribution temperature given by 10

$$V = A \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right).$$

The edges are kept at zero temperature and the plane faces are impervious to heat. Find the temperature at any point and time.

10. Attempt ***any two*** questions : **5x2=10**

(a) Solve 5

$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx.$

Where p and q have their usual meaning.

(b) Solve 5

$$\frac{y - z}{yz}p + \frac{z - x}{zx}q = \frac{x - y}{xy}$$

Where p and q have their usual meaning.

(c) Solve 5

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$$

Where symbols have their usual meaning.