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# BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

## Term-End Examination

## December, 2011

00862

## BME-001 : ENGINEERING MATHEMATICS-I

*Time : 3 hours* *Maximum Marks : 70*

***Note :*** *All questions are **compulsory**. Use of scientific calculator is allowed. Statistical tables are allowed.*

**1.** Attempt *any five* of the following : **5x4=20**

(a) Does the limit $\lim_{x \to 0} \frac{\sin^2 x}{x(1-\cos x)}$ exist ?

If yes, find the limiting value ?

(b) Find the value of 'b' for which the function

$$f(x)=\begin{cases} x^3+1, \text{ when } x<2 \\ bx+\frac{2}{x}, \text{ when } x \geq 2 \end{cases}$$

is continuous at $x=2$.

(c) Attempt any one of the following :

(i) Differentiate $\tan^{-1}\left(\sqrt{1+x^2}-x\right)$

(ii) Evaluate $\int_0^{\pi/2} \log(\sin x)\, dx$

(d) Use Cauchy's Theorem to show that $1+x<e^x$.

(e) If $y_1=\frac{x_2x_3}{x_1}, y_2=\frac{x_3x_1}{x_2}, y_3=\frac{x_1x_2}{x_3}$

find $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$

(f) Solve the differential equation

$\left(e^{-2\sqrt{x}}-y\right)dx=\sqrt{x}\, dy$.

2. Answer *any four* of the following : **4x5=20**

(a) A particle moves on the curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$, where $t$ is the time. Find the components of velocity and acceleration at time $t=1$.

(b) Find the directional derivative of $\phi=(x^2+y^2+z^2)^{-1/2}$ at the point p(3, 1, 2) in the direction of the vector $yz\hat{i}+zx\hat{j}+xy\hat{k}$.

(c) Prove that $\text{div}(\text{grad } r^n)=\nabla^2 r^n=n(n+1)r^{n-2}$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. Hence show that

$\nabla^2\left(\frac{1}{r}\right)=0$.

(d) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal.

(e) If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\,\hat{j} - 4x\hat{k}$, then evaluate $\iiint_v \nabla . \vec{F}\, dv$, where $v$ is bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$.

(f) Evaluate $\oint_c \vec{F} . d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$ and $c$ is the boundary of the triangle with vertices at (0, 0, 0), (0, 1, 0) and (1, 1, 0).

**3.** Answer *any five* of the following :

**5x3=15**

(a) Transform $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into a unit matrix by using elementary transformations.

(b) Find the inverse of the matrix 'M'

$$M = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Investigate for what values of $\lambda$ and $\mu$ do the system of equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$, have (i) No solution (ii) unique solution (iii) infinite solutions.

(d) If $A=\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, verify that A*A is a Hermitian matrix where A* is the conjugate transpose of A.

(e) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

(f) Establish the following identity using the properties of the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)\ (b-c)\ (c-a)\ (a+b+c)$$

(g) Solve the following system by Cramer's rule :

$x+2y-z=2$

$2x+3y+2z=7$

$-x+2y+3z=4.$

4. Answer *any three* of the following : **3x5=15**

(a) A problem of statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved ?

(b) Three urns A, B, C contain 6 red and 4 black balls, 2 red and 6 black balls and 1 red and 8 black balls respectively. An urn is chosen at random and a ball is drawn from the urn. If the ball drawn is red, find the probability that the ball was drawn from urn A.

(c) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of successes.

(d) A random sample of 400 male students have average weight of 55 kg. Can we say that the sample comes from a population with mean 58 kg with a variance of 9 kg ?

(e) A random sample of 16 values from a normal population is found to have a mean of 41.5 and standard derivation of 2.795. On this basis, is there any reason to reject the hypothesis that the population mean $\mu = 43$ ? Also find the confidence limits for $\mu$.