B.Tech. Civil (Construction Management)/ B.Tech. Civil (Water Resources Engineering) B.Tech. (Aero space Engineering)

BTCLEVI/BTMEVI/BTELVI/BTECVI/BTCSVI

Term-End Examination

December, 2011

02802

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of scientific calculator is **permitted**.*

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1. Answer *any five* of the following : **5x4=20**

(a) Evaluate the following limits, if they exist :

(i) $$\lim_{x \to 1} \frac{x^{-1/2} - 1}{x^{-1/5} - 1}$$

(ii) $$\lim_{x \to 0} \frac{1 - \cos x^2}{x^2 \sin x^2}$$

(b) A function f is defined as :

$$f(x) = \begin{cases} x^2 + 3x - 2, & \text{when } x < 1 \\ \dfrac{4}{x+1}, & \text{when } x \geq 1 \end{cases}$$

Is $f(x)$ continuous at $x = 1$? Justify.

(c) Find $\frac{dy}{dx}$ if $y^{\cos x} + (\sin x)^y = 3$.

(d) If $y = \ln\left(\sqrt{1+x^2} - x\right)$, prove that :

$$(1+x^2)y_2 + xy_1 = 0.$$

Hence find y_{n+2}, using Leibnitz's theorem.

(e) Show that there is no real number, λ for which the equation $x^2 - 4x + \lambda = 0$

has two distinct roots in $\left[\frac{1}{2}, \frac{3}{2}\right]$.

(f) If $u = \cos^{-1}\left(\frac{x}{y}\right) + \cot^{-1}\left(\frac{y}{x}\right)$,

show that : $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

2. Answer *any four* of the following : **4x4=16**

(a) Evaluate the following integrals :

(i) $\int_0^{\pi} \frac{x\, dx}{1 + \sin x}$

(ii) $\int e^x . \frac{1 + x}{(2 + x)^2}\, dx$

(b) Evaluate :

(i) $$\int_0^1 \frac{2x+3}{\sqrt{2x^2+6x+1}}\,dx$$

(ii) $$\frac{d}{dx}\int_{x^2}^{x^3} \sin t^2 \, dt$$

(c) Find the length of the arc of the parabola, $y^2 = 4ax$, cut off by its latus rectum.

(d) A river is 60 feet wide. The depth 'd' in feet at a distance x feet from one bank is given as :

x :	0	10	20	30	40	50	60
d :	0	3	8	14	10	7	3

Find approximately the area of cross-section using Simpson's Rule.

(e) Solve the differential equation :

$$\frac{dy}{dx} + xy = y^2 e^{x^2/2} \ln x$$

3. Answer *any four* of the following : **4x4=16**

(a) Find div curl $\vec{F}$ where

$$\vec{F} = x^2 y \hat{i} + xz \hat{j} + 2yz \hat{k}.$$

(b) Show that the vector field given by :

$$\vec{F} = x^2y^3z^3\hat{i} + x^3y^2z^3\hat{j} + x^3y^3z^2\hat{k}.$$

is conservative. Also find a scalar potential u such that $\vec{F} = \text{grad } u$.

(c) In what direction from the point (3, 2, 1) is the directional derivative of :

$$\phi = \phi(x, y, z) = x^2 - 2y^2 + 4z^2$$

a maximum ? Also find the value of this maximum directional derivative.

(d) If the field of force is given by :

$$\vec{F} = y\hat{i} + z\hat{j} + x\hat{k},$$

find the work done in moving a particle once round the circle, $x^2 + y^2 = 4$, $z = 0$.

(e) If $\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ be a continuous vector point function, verify that it is solenoidal and find the function f such that $\vec{F} = \text{curl } f$.

4. Answer *any three* of the following : **3x6=18**

(a) Show that :

$V = \{(a, b, c, d) \mid a + b = 0 \text{ and } c = d\}$

is a subspace of R^4. Also find its dimension.

(b) Solve the following equations by matrix method :

$$x - y + z = 1$$

$$2x + 6y + 3z = 11$$

$$3x - y - z = 1$$

(c) Prove that :

$$\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$$

(d) Find the eigen values and the eigen vectors of the matrix :

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$