# MCA (Revised) 

Term-End Examination
December, 2011
MCS-013 : DISCRETE MATHEMATICS

Time : 2 hours
Maximum Marks : 50
Note: Question number 1 is compulsory. Attempt any three questions from the rest.

1. (a) If there are 12 persons in a party, and if each 3 two of them shake hands with each other how many hand shakes happen in the party ?
(b) Prove that $A-B=A \Leftrightarrow A \cap B=Q$
(c) Let $R$ be the binary relation defined as
$R=\left\{(a, b) \in R^{2} \mid a-b \leq 3\right\}$
Determine whether R is reflexive, symmetric antisymmetric or transitive.
(d) What is a propositional function? Write propositional function for following statement.

Always there are some students in a class who are hardworking.
(e) Represent the following argument symbolically and determine whether the argument is valid?
"If today is Children's day then today is Pt.Jawaharlal Nehru's birthday". "If today is Pt.Jawaharlal Nehru's birthday then today is $14^{\text {th }}$ Nov".
Hence "If today is Children's day then today is $14^{\text {th }}$ Nov.
(f) Simplify the Boolean function

$$
\mathrm{B}\left(x_{1}, x_{2}, x_{3}\right)=\left[\left(x_{1} \wedge x_{2}\right) \vee\left(\left(x_{1} \wedge x_{2}\right) \wedge x_{3}\right)\right] \vee\left(x_{2} \wedge x_{3}\right)
$$

2. (a) Using mathematical induction method, show that :
$1^{3}+2^{3}+3^{3}+\ldots . .+n^{3}=\left[\frac{n(n+1)}{2}\right]^{2}$
(b) Find the number of different messages that can be represented by sequences of 4 dashes and 3 dots.
(c). If I be the set of integers, find whether $f: I \rightarrow$ I defined by $f(x)=x^{3}$ is one-one onto or both.
3. (a) Construct a logic circuit represented by the 4 Boolean expression.
$\left(x_{1}^{\prime} \wedge x_{2}\right) \vee\left(x_{1} \vee x_{3}^{\prime}\right) \wedge\left(x_{2} \vee x_{3}\right)$
where $x_{i}(1 \leq i \leq 3)$ are assumed to be inputs to that circuitry.
(b) Verify that the proposition $\mathrm{P} \vee \sim(\mathrm{P} \wedge \mathrm{Q})$ is a tautology.
(c) A valid Computer password consists of nine characters, the first of which is the digit 1,5 or 7 the third character is either a \# or a $\$$ and the remaining a english alphabet or a digit. Find how many different passwords are possible?
4. (a) Let $I$ be the set of all integers. Let $R$ be a relation on $I$, defined by $\mathrm{R}=\{(x, y): x-y$ is divisible by $6 \forall x, y \in \mathrm{I}\}$ Show that R is an equivalence relation.
(b) Give the geometric representation of 3 $\{3\} \times \mathbb{R}$.
(c) There are 15 points in a plane, no three of 4 which are collinear. Find the number of straight lines formed by joining them.
5. (a) If 100 bulbs are placed in 15 boxes. Show 3 that two of the boxes must have the same number of bulbs.
(b) If $f(x)=x^{2}$ and $g(x)=x+1$, then find $(f \circ g) x \quad 4$
and (gof)x.
(c) Explain with reason, whether or not 3
(i) the collection of all good teachers is a set.
(ii) the set of points on a line is finite.
