# ADCA / MCA (II YEAR) 

Term-End Examination

00620
December, 2011

## CS-51 : OPERATIONS RESEARCH

Time: 3 hours
Maximum Marks : 75
Note: Question number 1 is compulsory. Attempt any three more questions from questions numbered 2 to 5.

1. (a) A company produces two types of Alloys
$\mathrm{A}_{1}$ and $\mathrm{A}_{2}$. Alloy $\mathrm{A}_{1}$ contains $50 \%$ copper and $50 \%$ zinc where as Alloy $\mathrm{A}_{2}$ contains $75 \%$ copper and $25 \%$ zinc. Net profits are Rs. 6000 per ton on $A_{1}$ and Rs. 5000 per ton on $A_{2}$. The daily supply to the company of copper is 15 tons and that of zinc is 10 tons. Formulate this problem as a linear programming problem for determining the number of tons of Alloys $A_{1}$ and $A_{2}$ should be produced so as to maximize the daily net profit (No need to solve this problem)
(b) Three Jobs $\mathrm{J}_{1}, \mathrm{~J}_{2}, \mathrm{~J}_{3}$ are to be assigned to three workers $W_{1}, W_{2}, W_{3}$. Each of the jobs is to be assigned to one and only one worker. The payments to different workers for doing different jobs differ and are shown in the table below :

| Job | Workers |  |  |
| :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |
| 1 | 40 | 30 | 20 |
| 2 | 10 | 50 | 40 |
| 3 | 30 | 40 | 40 |

Find the optimal assignment which minimizes the total payment to workers for doing the jobs. Also write the total payment for the optimal assignment.
(c) Explain the terms "Non-linear 4 Programming" and "Integer Programming".
(d) What is simulation? Indicate the reasons 4 for using it.
(e) Explain the terms : Inventory, Ordering 4 cycle, lead time, stock out cost.
(f) In a TV repair shop with one server, TVs 8 arrive at the shop in a Poisson fashion at the average rate of 2 TVs per hour while the repair time of TVs has an exponential distribution with the average repair rate of 4 TVs per hour. Compute the following :
(i) The probability that these are

$$
\mathrm{n}(\mathrm{n}=0,1,2, \ldots) \mathrm{TVs} \text { at any time. }
$$

(ii) Average number of TVs at the shop.
(iii) The probability that the server is idle.
2. (a) Apply simplex method to solve the problem LPP :

$$
\text { Minimize } x_{0}=x_{1}-x_{2}
$$

Subject to

$$
\begin{aligned}
& 2 x_{1}+x_{2} \leq 4 \\
& 2 x_{1}-x_{2} \leq 2 \\
& x_{1}, x_{2} \geqslant 0
\end{aligned}
$$

(b) Describe a queueing system. Explain the terms : Server, Kendall's notation, time spent by a customer in the queueing system in the content of a queueing system.
3. (a) Write Kuhn-Tucker condition for the 7 problem given as follows:

$$
\text { Minimize } x_{0}=x_{1}^{2}+x_{2}^{2}-2 x_{1}-2 x_{2}+2
$$

Subject to

$$
\begin{aligned}
& x_{1}+2 x_{2} \leq 2 \\
& x_{1}, x_{2} \geqslant 0
\end{aligned}
$$

After this, obtain the linear programming problem with the restricted basis conditions, whose optimal solution would yield the optimal solution of the given problem.
(b) Using the North West Corner Method to find an initial basic feasible solution, solve the transportation problem minimizing the total cost of transportation given as follow :

| Destinations |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | Availability $a_{i}$ |
| 40000 | $\mathrm{S}_{1}$ | 1 | 3 | 6 | 10 |
|  | $\mathrm{S}_{2}$ | 4 | 1 | 10 | 30 |
|  | $\mathrm{S}_{3}$ | 2 | 1 | 6 | 40 |
|  | Requirement $b_{j}$ | 20 | 25 | 35 |  |

4. (a) Explain the terms: unbounded solution and multiple solutions in the content of linear programming problem. Then write two simple linear programming problems in two decision variables out of which one has an unbounded solution and the other one has multiple solutions.
(b) For the primal problem

$$
\begin{gathered}
\text { Maximize } z=3 x_{1}-4 x_{2} \\
\text { subject to }-x_{1}+3 x_{2}=7 \\
-x_{1}+2 x_{2}=3 \\
x_{1}, x_{2} \geqslant 0
\end{gathered}
$$

Write its dual in two variables both of which are unrestricted in sign. Also write the dual of the dual of the above primal problem and show that it is identical with the primal problem.
5. (a) Find the saddle point solution mentioning the optimal strategies for the players $X$ and $Y$ together with the value of the game. The payoff matrix for the player $X$ is as given as follows:

|  | Strategies for Player Y |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $Y_{1}$ | $Y_{2}$ | $Y_{3}$ | $\mathrm{Y}_{4}$ | $Y_{5}$ |
|  | $\mathrm{X}_{1}$ | 13 | 3 | -1 | 4 | 3 |
|  | $\mathrm{X}_{2}$ | 2 | 7 | 1 | 3 | 4 |
|  | $\mathrm{X}_{3}$ | 6 | 10 | 0 | 8 | 6 |

(b) Use dynamic programming technique to 9 find the point $(x, y)$ in the first quadrant on the straight line $3 x+2 y=6$ nearest the origin. Also write the distance of the point on the line nearest the origin.

