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CS-601

01651

BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

December, 2011

CS-601 : DIFFERENTIAL AND INTEGRAL CALCULUS WITH APPLICATIONS

Time : 2 hours *Maximum Marks : 75*

Note : *Question no. **1** is **compulsory**. Attempt **any three** more questions from question Nos. **2 to 6**. Use of calculator is permitted.*

1. (a) Select the correct answer from the four given alternatives for each part given below.

$1 \times 6 = 6$

(i) If $x = at^2$, $y = 2at$, then $\frac{dy}{dx}$ is equal to :

(A) t (B) at

(C) $\frac{1}{t}$ (D) $\frac{1}{at}$

(ii) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta}$ is :

(A) 1 (B) 0

(C) ∞ (D) None of these

(iii) The equation of the tangent to the curve $y = 2x^2 - 3x - 1$ at $(1, -2)$ is

(A) $x + y + 1 = 0$

(B) $x - y - 3 = 0$

(C) $x + 4y = 1$

(D) None of the above

(iv) If $y = e^{\cos x}$, then $\frac{dy}{dx}$ is

(A) $e^{\cos x}$ (B) $\sin x \; e^{\cos x}$

(C) $-\sin x \; e^{\cos x}$ (D) $e^{\cos x - 1}$

(v) $\int (x^2 + 1)^3 .2x \, dx$ is :

(A) $\frac{1}{6}(x^2 + 1)^6 + C$ (B) $\frac{1}{2}(x^2 + 1)^2 + C$

(C) $\frac{1}{4}(x^2 + 1)^4 + C$ (D) $\frac{1}{8}(x^2 + 1) + C$

(vi) $\lim_{x \to 0} x \cos x$

(A) 0 (B) 1

(C) ∞ (D) None of these

(b) Fill in the blanks: **1 × 6 = 6**

(i) $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} =$ _____

(ii) The point of maximum value of the function $f(x) = \sin 2x$ in the interval $\left[0, \frac{\pi}{2}\right]$ is $x =$ ____

(iii) The value of the definite integral $\int_0^{\pi/2} (5\sin x + 2\cos x)dx$ is _____

(iv) $\lim_{x \to o} \frac{e^x - 1}{x}$ *is* _____

(v) The minimum value of the function $f(x) = x^3 - 3x$ in the closed interval [0, 2] is ____

(vi) $\int (4x+2)\sqrt{x^2+x+1}\ dx =$ _____

(c) A function $f(x)$ is defined as **3**

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$$

Find the limit of $f(x)$ as $x \to 3$, and prove that the function is continuous for $x = 3$.

(d) If $y = \frac{\sin^2 x}{1+\cos^2 x}$, prove that $\frac{dy}{dx} = \frac{2\sin 2x}{[1+\cos^2 x]^2}$ **3**

(e) Evaluate $\int_0^{\pi} (2\cos x - x)\, dx$ **3**

(f) Show that the height of an open cylinder of given surface, that can contain maximum water, is equal to radius of its base. **3**

(g) If $z = f(x+ct) + \phi(x-ct)$, prove that **3**

$$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$

(h) Verify Rolle's theorem for the function. **3**

$$f(x) = \sin x + \cos x \text{ on } \left[0, \frac{\pi}{2}\right]$$

2. (a) Evaluate $\int x^2 \cos x \, dx$ **3+4+4+4**

(b) Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \, dx$

(c) Evaluate $\int_0^{\pi/2} \frac{\cos 2x}{\cos x + \sin x} \, dx$

(d) A cube is expanding in such a way that its edge is changing at a rate of 5 cm/sec. Compute the rate of change of its volume when its edge is 4 cm long.

3. (a) Prove that the common area between two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **5+5+5**

(b) Solve the differential equation (Any one)

(i) $x\frac{dy}{dx} + y = x^3y^6$

(ii) $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$

(c) The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

Using Simpson's $\frac{1}{3}$rd rule, Estimate approximately the distance covered in 20 minutes.

4. (a) Examine the differentiability of $f(x)$ at $x=2$. **5+5+5**

$$f(x)=1+x \quad x \leq 2$$

$$= 5-x \quad x>2$$

(b) If $y=x^{x^x}$, then find $\frac{dy}{dx}$

(c) If $y=\sin(m \sin^{-1} x)$, prove that :

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0$$

5. (a) Show that the curve $x^3 - 3xy^2 = 2$, and

$3x^2y - y^3 = 2$ cut orthogonally. **5+5+5**

(b) Find the volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius R.

(c) Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.

6. (a) Is the function **5+5+5**

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{when } x \neq 1 \\ 2 & \text{when } x = 1 \end{cases}$$

continuous at $x = 1$? Explain your answer.

(b) Find the equations of the tangent and normal to the curve.

$4x^3 + 4xy + y^2 = 4$ at (0, 2)

(c) A right circular cone has a given curved surface 'S'. Show that, its volume will be maximum when the ratio of the height to the base radius is $\sqrt{2} : 1$.
