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MMT-002

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2010

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 5 is **compulsory**. Answer **any three** questions from question Nos. **1** to **4**. Use of calculators is **not** allowed.*

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1. (a) Check whether the matrices $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ are similar or not. **2**

(b) Find the spectral decomposition of the operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by **3**

$$T(x, y, z) = (6x - y - 2z, -x + 5y - z, -2x - y + 6z)$$

2. (a) Find a QR decomposition of the matrix **2**

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

(b) Consider the predator - prey system given by 3

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{bmatrix} 1.1 & 0.4 \\ -0.104 & 0.5 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix}$$

Where x_k and y_k are populations of the predators and the prey, respectively, at time k. Find the long term behaviour of the population vector. $\begin{bmatrix} x_k \\ y_k \end{bmatrix}$

3. (a) Compute the Jordan form of the matrix 3½

$$A = \begin{bmatrix} 4 & -2 & 0 & 1 \\ 5 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix}$$

(b) Find a least squares solution of the system $Ax = b$, where 1½

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

4. (a) Find a basis of each eigenspace of the operator $T : \mathbb{R}^3 \to \mathbb{R}^3$, defined by $T(x,y,z) = (2x + y,\ y - z,\ 2y + 4z)$. Is T diagonalizable ? Give reasons for your answer. 3

(b) Find the singular values of the matrix **2**

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}.$$

5. Which of the following statements are true, and which are false ? Give reasons for your answers. **10**

(i) A nilpotent operator is diagonalisable.

(ii) The QR - decomposition of any non-singular matrix is unique.

(iii) There is a 2x2 unitary matrix with eigen values 2 and $\frac{1}{2}$.

(iv) If the eigen values of A ε $\mathbf{M}_2$ (C) are 3,2, set $(e^A) = e^6$.

(v) If A ε $\mathbf{M}_n$ (C) such that tr $(AA^*) = 0$, then $A = 0$.