BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination December, 2010

BME - 015: ENGINEERING MATHEMATICS-II

Time: 3 hours Maximum Marks: 70

Note: Answer any ten of the following questions. All questions carry equal marks. Use of calculator is permitted.

- 1. Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \left[\sqrt{(n^2 + 1)} n \right]$
- 2. Test for convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{4.7....(3n+1)}{1.2...n} x^n$
- 3. For a function f(x) defined by $f(x) = |x|, -\pi, -\pi < x < \pi,$ obtain a fourier series.

$$x^{2} = \frac{\pi^{2}}{3} + 4 \sum_{n=1}^{\infty} (-1)^{n} \frac{\cos nx}{n^{2}} - \pi < x < \pi,$$
Hence show that $\sum \frac{1}{n^{2}} = \frac{\pi^{2}}{6}$,

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5. If $2 \cos \theta = x + \frac{1}{x}$, and $2 \cos \phi = y + \frac{1}{y}$, show that one of the value of $x^m y^n + \frac{1}{x^m y^n}$ is $2 \cos (m\theta + n\phi)$.

6. If
$$\tan \log (x + iy) = a + ib$$
,

where $a^2 + b^2 \ne 1$, show that

 $\tan \log (x^2 + y^2) = \frac{2a}{1 - a^2 - b^2}$

7. Show that the polar form of Cauchy - Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \qquad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$$

Deduce that
$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$$

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- 8. Find the bilinear transformation which maps the points Z = 1, i, -1 into the points $\omega = 2$, i, -2 respectively.
- 9. Expand $f(Z) = \frac{1}{(Z-1)(Z-2)}$ in the region |Z|>2
- 10. Determine the poles of the following functions and the residue at each pole: $\frac{Z^2 + 1}{Z^2 2Z}$
- 11. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What would be the value of N after 1½ hours?
- 12. Solve $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$
- 13. Solve (mz ny) p + (nx lz)qw = ly mx
- 14. Solve $\frac{\partial^2 Z}{\partial x^2} + 3 \frac{\partial^2 Z}{\partial x \partial y} + 2 \frac{\partial^2 Z}{\partial y^2} = x + y$

15. Find the solution of

$$\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$$

for which u (0, t) = u(l, t) = 0

 $u(x, 0) = \sin \frac{\pi x}{l}$ by method of variables separation.

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