BME-001

01475

BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

December, 2010

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

1. Answer *any five* of the following : 5x4=20

(a) Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4 \quad , & x = 1 \\ b - ax, & x > 1 \end{cases}$

and if $\lim_{x \to 1} f(x) = f(1)$, what are the possible values of a and b ?

(b) Examine the continuity and differentiability of $f(x)$ at $x = 2$.

$$f(x) = 1 + x, \quad x \leq 2$$
$$= 5 - x, \quad x > 2$$

(c) If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)\, y_{n+2} - (2n+1)\, x\, y_{n+1} + (m^2 - n^2)\, y_n = 0$.

(d) If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$,

find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$.

(e) Evaluate (any one)

(i) $\int_1^2 x^2 \log x \, dx$

(ii) $\int_0^{\pi/3} \frac{\sin x}{(1 + \cos x)^2} \, dx$

(f) Solve *any one* of the following :

(i) $\frac{dy}{dx} = e^{2x - 3y} + 4x^2 . e^{-3y}$

(ii) $(x + y + 1)^2 \frac{dy}{dx} = 1$

2. Answer ***any four*** of the following : **4x4=16**

(a) What is the unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$, if $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$, and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

(b) A particle is acted on by constant forces $2\hat{i} + \hat{j} - \hat{k}$, $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} + \hat{j} + 5\hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $6\hat{i} + 3\hat{j} + \hat{k}$. Find the work done.

(c) Find div $\vec{F}$ where

$$\vec{F} = \text{grad}\ (x^3 + y^3 + z^3 - 3\ xyz)$$

(d) Show that the vector

$$\vec{V} = (x + 3y)\ \hat{i} + (y - 3z)\ \hat{j} + (x - 2z)\ \hat{k}$$

is solenoidal.

(e) Use divergence theorem to show that

$$\oint_s \nabla\ r^2 . d\vec{S} = 6\ V$$

where S is any closed surface enclosing a volume V and $r^2 = x^2 + y^2 + z^2$.

(f) The position vector of a moving particle is given by $r\ (t) = t^3\hat{i} + t\hat{j} + t^2\hat{k}$.

Determine the velocity, speed and acceleration of the particle in the direction of the motion.

3. Answer *any six* of the following : **6x3=18**

(a) Express the following matrix as the sum of a symmetric and a skew - symmetric matrix.

$$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$$

(b) Given :

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

Find x, y, z, and w.

(c) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$

Obtain the product AB and explain why BA is not defined.

(d) Prove that the product of two matrices

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and } \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is a null matrix when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

(e) Find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

(f) Solve the system of the following equations by Cramer's rule :

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

(g) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(h) Find the eigen values of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

4. Answer *any four* of the following : **4x4=16**

(a) In a relay race there are five teams A, B, C, D and E.

(i) What is the probability that A, B and C finish first, second and third, respectively.

(ii) What is the probability that A, B and C are first three to finish (in any order)

(Assume that all finishing orders are equally likely).

(b) In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing atleast one of them is 0.95. What is the probability of passing both ?

(c) The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now 60, atleast 7 will live to be 70 ?

(d) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals, the number of persons suffering a bad reaction are :

(i) exactly 3

(ii) more than 2 individuals

(iii) none

(iv) more than one individual.

(e) The lifetime of radio tubes manufactured in a factory is known to have an average value of 10 years. Find the probability that the lifetime of a tube taken randomly

(i) exceeds 15 years,

(ii) is less than 5 years,

assuming that the exponential probability law is followed.

(f) The mean life time of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. The company claims that the average life of the bulbs produced by it is 1600 hours. Using the level of significance of 0.05, is the claim acceptable ?