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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) B.Tech. (Aerospace Engineering)

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Term-End Examination

December, 2010

ET-102 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

Note : Answer any ten questions. All questions carry equal marks. Use of calculator is allowed.

1. Test the series

$\Sigma\left(\sqrt[3]{n^3+1} - n\right)$ for its convergence or divergence

2. Find the Fourier series to represent
$f(x) = x - x^2, -\pi < x < \pi$.
Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$$

3. Find the bilinear transformation which maps the points $z = 0, -1, \alpha$ into the points $w = -1, -2-i, i$.

4. Show that the polar form of Cauchy-Riemann equations are :

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial \upsilon}{\partial \theta}, \quad \frac{\partial \upsilon}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Deduce that :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

5. Find the general solution of the partial differential equation

$x^2 (y-z)\, p + y^2 (z-x) q = z^2 (x-y)$

6. If $2\cos\theta = x + \frac{1}{x}$ and

$$2\cos\Phi = y + \frac{1}{y},$$

show that one of the values of

$$\frac{x^m}{y^n} + \frac{y^n}{x^m} \text{ is } 2\cos(m\theta - n\Phi)$$

7. Test for convergence of the series

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \ldots + \frac{2^n - 2}{2^n + 1}x^{n-1} + \ldots,$$

(for $x > 0$)

8. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What would be the value of N after 1½ hours ?

9. Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

10. (a) Find the Laplace transforms of
$f(t) = t, \quad t \geq 0,$
(b) Find the Laplace transforms of sin*h* at

11. Find the inverse Laplace transforms of

$$\frac{s^2 - 3s + 4}{s^3}$$

12. Determine the poles of the function

$$f(z) = \frac{z^2 + 1}{z^2 - 2z}$$ and the residue at each pole.

13. Solve by the method of Laplace transforms, the equation
$y''' + 2y'' - y' - 2y = 0,$
given $y(0) = y'(0) = 0$, and $y''(0) = 6$

14. Apply Hurwitz - Routh criterion to determine the stability of the systems whose characteristic equations are given by

(a) $s^4 + 2s^3 + s^2 + 3s + 2 = 0$

(b) $s^5 - 2s^4 + 2s^3 + 4s^2 - 11s - 10 = 0$

15. Establish the differential equation describing the behaviour of the parallel or one - node - pair, electric circuit shown below.