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ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) B.Tech (Aero space Engineering)

03451

Term-End Examination

December, 2010

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are compulsory. Use of calculator is permitted.*

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1. Answer **any five** of the following : **5x4=20**

(a) (i) State and prove De Moivre's formula for positive integral value.

(ii) Find the modulus and the argument of $z = -i/(2+2i)$.

(b) Find the value of b for which the function

$$f(x) = \begin{cases} x^3+1, & x<2 \\ bx+\dfrac{2}{x}, & x \geq 2 \end{cases}$$

is continuous, at $x = 2$

(c) If $x = \sin t$, $y = \cos pt$, then prove that

$$(1-x^2)\, y^{(2)} - xy^{(1)} + p^2 y = 0$$

(d) Using the mean value theorem show that

$$3+\frac{1}{28}<(28)^{1/3}<3+\frac{1}{27}$$

(e) Show that

$$\text{Sin}\left[\log_e(x^2+2x+1)\right]=2x-x^2-\frac{2}{3}x^3+\frac{3}{2}x^4-\frac{5}{3}x^5+\ldots$$

(f) Find the maximum and minimum values of

$$xy+27\left(\frac{1}{x}+\frac{1}{y}\right).$$

(g) Find $\frac{dy}{dx}$ when $y=\tan^{-1}\frac{\cos x-\sin x}{\cos x+\sin x}$.

2. Answer **any four** of the following : **4x4=16**

(a) Evaluate (any one)

(i) $\int_0^1 x^4\left(1-x^2\right)^{3/2}dx$

(ii) $\int_2^5 \log_e x\,dx$

(b) For the cycloid $x=a(t+\sin t)$, $y=a(1-\cos t)$ obtain the area between its base and the portion of the curve from cusp to cusp.

(c) Find the total length of the curve given by

$x = a\cos^3\theta,\ y = a\sin^3\theta.$

(d) Find the volume of solid obtained by revolving the limacon

$r = a + b\cos\theta$

about the initial line.

(e) Following table gives the speed v meters per second of a car, *t* seconds after it starts from rest. Using Simpson's rule, find the distance travelled in two minutes.

t :	12	24	36	48	60	72	84	96	108	120
v :	3.60	10	18.9	21.6	19	10	5.40	4.50	5.40	9.00

(f) Solve (any one)

(i) $x\,dx + y\,dy + 2(x^2 + y^2)\,dx = 0$

(ii) $x\frac{dy}{dx} + y = 3x^3y^3.$

3. Answer any four of the following : 4x4=16

(a) Show that the vectors

$\vec{a} = 3\sqrt{3}\,\hat{i} - 3\,\hat{j},\ \ \vec{b} = 6\,\hat{j},\ \ \vec{c} = 3\sqrt{3}\,\hat{i} + 3\,\hat{j}$

form the sides of an equilateral triangle.

(b) Find the angle between the surfaces

$x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.

(c) If $\vec{a}$ and $\vec{r}$ are constant vectors, show that

$$\nabla \times \left\{ \vec{a} \times \left(\vec{r} \times \vec{r} \right) \right\} = \vec{a} \times \vec{r}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(d) Show that $\iint_S \vec{F} \cdot \hat{N}\, ds = \frac{3}{8}$, where

$\vec{F} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ and S is the portion of

the surface of the sphere $x^2 + y^2 + z^2 = 1$ in

the first octant.

(e) Verify Green's Theorem for

$$\oint_c \left[\left(3x^2 - 8y^2\right) dx + 2y(2 - 3x)\, dy \right],$$

where c is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$

(f) Show that the vector function

$$\vec{F} = x(y - z)\,\hat{i} + y(z - x)\,\hat{j} + z(x - y)\,\hat{k}$$

is solenoidal and find the function f such that $\vec{F} = \text{curl}\, f$.

4. Answer any six of the following : **6x3=18**

(a) If the matrix $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then show that

$$a = \pm\frac{1}{\sqrt{2}},\ b = \pm\frac{1}{\sqrt{6}} \text{ and } c = \pm\frac{1}{\sqrt{3}}$$

(b) Show that

$$\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

(c) Find the inverse of

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 4 \\ 1 & 5 & -2 \end{bmatrix}$$

if it exists.

(d) Prove that for every square matrix A, the matrix $(A + A^T)$ is symmetric.

(e) State Cayley - Hamilton Theorem and using it find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(f) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & -1 & 5 \\ 5 & 6 & 0 \end{bmatrix}$

has atleast one negative eigen value.

(g) Define the diagonal form of a matrix. Obtain the same for

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

(h) Compute e^A , where $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$

(i) Transform the quadratic form

$Q = 17x_1^2 - 30x_1 x_2 + 17x_2^2 = 128$ to the canonical form. Also find the matrix of transformation and the transformation.
