

**POST GRADUATE DIPLOMA IN  
APPLIED STATISTICS (PGDAST)**

**Term-End Examination**

**June, 2015**

00658

**MST-003 : PROBABILITY THEORY**

*Time : 3 hours*

*Maximum Marks : 50*

**Note :**

- (i) *Attempt all questions. Questions no. 2 to 5 have internal choices.*
- (ii) *Use of scientific calculator is allowed.*
- (iii) *Use of Formulae and Table Booklet for PGDAST is allowed.*
- (iv) *Symbols have their usual meanings.*

1. Which of the following statements are *True* or *False* ? Give reasons in support of your answer.

$5 \times 2 = 10$

- (a) For any event A,  $P(A) + P(\bar{A}) < 1$ , where  $\bar{A}$  denotes complement of A.
- (b) If odds in favour of an event A are 3 : 4, then the probability of occurrence of A will be  $\frac{4}{7}$ .
- (c) If  $F(x)$  denotes probability distribution function of a random variable X, then  $F(x) = P[X > x]$ .

- (d) If a random variable  $X$  follows binomial distribution and  $p$  represents the probability of success, then  $p$  varies from trial to trial.
- (e) A random variable  $X$  follows normal distribution having p.d.f.

$$f(x) = \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{32}(x-60)^2}, -\infty < x < \infty$$

The variance of the random variable  $X$  is 4.

2. (a) Find the probability of getting 53 Sundays in a randomly selected non-leap year. 5
- (b) A class has 15 students whose ages are 14, 17, 15, 21, 19, 20, 16, 18, 20, 17, 14, 17, 16, 19 and 20 years, respectively. One student is chosen at random and the age of the selected student is recorded. What is the probability that
- (i) the age of the selected student is divisible by 3,
  - (ii) the age of the selected student is more than 16, and
  - (iii) the selected student is eligible to cast the vote where the minimum age to cast the vote is 18 years? 5

**OR**

(a) There are 40 pages in a book. A page is opened at random. Find the probability that the number of the opened page is a multiple of 3 or 5. 5

(b) An insurance company insured 1000 scooter drivers, 3000 car drivers and 6000 truck drivers. The probabilities that the scooter, car and truck drivers meet an accident are 0.02, 0.04, 0.25, respectively. One of the insured persons meets with an accident. What is the probability that he is a

(i) car driver,

(ii) truck driver ? 5

3. (a) For the following probability distribution of a discrete random variable X,

X	0	1	2	3	4	5
p(x)	0	C	C	2C	3C	C

find

(i) the constant C,

(ii)  $P[X \leq 3]$ , and

(iii)  $P[1 < X < 4]$ . 1+2+2

(b) Two discrete random variables X and Y have

$$P[X = 0, Y = 0] = \frac{2}{9}, P[X = 0, Y = 1] = \frac{1}{9},$$

$$P[X = 1, Y = 0] = \frac{1}{9} \text{ and } P[X = 1, Y = 1] = \frac{5}{9}.$$

Examine whether X and Y are independent. 5

**OR**

- (a) Let  $X$  and  $Y$  be two random variables. Then for

$$f(x, y) = \begin{cases} k(2x + y), & 0 < x < 1, \quad 0 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$$

to be a joint density function, what must be the value of  $k$  ?

5

- (b) For the following probability distribution :

X	-2	-1	0	1	2
p(x)	0.15	0.30	0	0.30	0.25

find

- (i)  $E(X)$   
(ii)  $E(2X + 3)$ ,  
(iii)  $E(X^2)$ , and  
(iv)  $E(4X - 5)$ .

1+1+2+1

4. (a) The chances of catching cold by workers working in an ice factory during winter are 25%. What is the probability that out of 5 workers 4 or more will catch cold ?

5

- (b) It is known that the number of heavy trucks arriving at a railway station follows the Poisson distribution. If the average number of truck arrivals during a specified period of an hour is 2, find the probabilities that during a given hour

- (i) no heavy truck arrives,  
(ii) at least two trucks arrive.

5

**OR**

(a) Obtain the mean and variance of the discrete uniform distribution for the random variable which denotes, "the number on a ticket drawn randomly from an urn containing 10 tickets numbered 1 to 10". Also obtain the expected frequencies, if the experiment is repeated 150 times. 5

(b) An unbiased dice is thrown until 6 appears. What is the probability that it must be thrown more than five times ? 5

5. (a) Write any 5 chief characteristics of the normal distribution. 5

(b) If the random variable  $X$  is normally distributed with mean 80 and standard deviation 5, then find  $P[60.5 < X < 90]$ . 5

**OR**

(a) If  $X$  is uniformly distributed with mean 2 and variance 12, find  $P[X < 3]$ . 5

(b) Telephone calls arrive at a switchboard at an average rate of 2 per minute. Let  $X$  denote the waiting time in minutes until the 4<sup>th</sup> call arrives and follows gamma distribution. Write the probability density function of  $X$ . Also find its mean and variance. 5