

**M.Sc. (MATHEMATICS WITH APPLICATIONS
IN COMPUTER SCIENCE)**

M.Sc. (MACS)

Term-End Examination

00918

June, 2015

MMTE-005 : CODING THEORY

Time : 2 hours

Maximum Marks : 50

(Weightage : 50%)

Note : Answer any *five* questions from questions no. 1 to 6. Use of calculator is *not* allowed.

1. (a) Define Hamming distance and Hamming weight. Illustrate them with the help of examples. 3
- (b) Define a self-orthogonal code and give an example of self-orthogonal code. 3
- (c) Write down the generator matrix of the Reed-Muller code $R(2, 4)$. 4

2. (a) Check whether the following code is a cyclic code or not :
 $\{000, 011, 101, 110\}$
Justify your answer. 2

(b) Find the dual code of a code C whose

$$\text{generator matrix is } G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

over F_3 .

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(c) Let $f(x) = x^3 + 2x^2 + 1 \in F_3[x]$.

(i) Prove that $f(x)$ is irreducible over F_5 .

(ii) Check whether the polynomial is primitive or not.

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3. (a) Factorize $x^9 - 1$. If α is a primitive 9th root of unity in F_{64} , show that

$$M_\alpha(x) = x^6 + x^3 + 1.$$

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(b) State the MacWilliams equations. Also find the weight enumerator of the binary code with generator matrix

5

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

4. (a) Construct the syndrome table for the code

$$\text{with generator matrix } \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

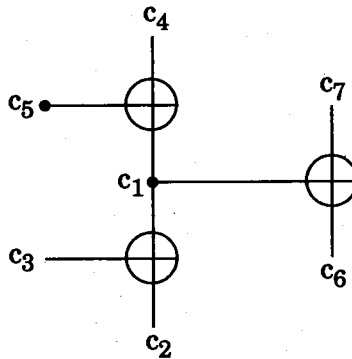
3

(b) Let $n = \frac{q^r - 1}{q - 1}$, where $\gcd(r, q - 1) = 1$. Let C be the narrow sense BCH code with defining set $T = C_1$. Prove that C is the Hamming code $H_{q,r}$.

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5. (a) Let $p \neq 3$ be an odd prime. Show that 3 is a square modulo p , if and only if $p \equiv \pm 1 \pmod{12}$. 6
- (b) If $f(x) \in \mathbf{Z}_4[x]$ is a basic irreducible polynomial, prove that $f(x)$ is a primary polynomial. 4

6. (a) Draw the State diagram and the Trellis diagram for the convolutional code with generator matrix $G = [1 + D^2 \quad 1 + D + D^2]$. 5
- (b) Find the parity check matrix for code with Tanner graph given below : 3



- (c) Give an example of \mathbf{Z}_4 -linear code of length 2 with 16 elements. 2