# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) <br> M.Sc. (MACS) 

Term-End Examination
00716
June, 2015

## MMTE-003 : PATTERN RECOGNITION AND IMAGE PROCESSING

Time : 2 hours
Maximum Marks : 50
Note: Attempt any five questions. All questions carry equal marks. Use of calculator is not allowed.

1. (a) Consider the following 2 -bit image of size $5 \times 5$.
$\left[\begin{array}{lllll}0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 3 & 2 & 2 \\ 2 & 3 & 1 & 0 & 0 \\ 3 & 3 & 2 & 2 & 0 \\ 1 & 2 & 3 & 0 & 1\end{array}\right]$

Compute histogram components and second order moments of the image before and after histogram equalization.
(b) Two images $f(x, y)$ and $g(x, y)$, have histograms $h_{f}$ and $h_{g}$ respectively. Give the conditions under which you can determine the histograms of the following:
(i) $\mathrm{f}(\mathrm{x}, \mathrm{y})+\mathrm{g}(\mathrm{x}, \mathrm{y})$
(ii) $\mathrm{f}(\mathrm{x}, \mathrm{y})-\mathrm{g}(\mathrm{x}, \mathrm{y})$
2. (a) Show that if a filter transfer function $H(u, v)$ is real and symmetric, then corresponding spatial domain filter $\mathrm{h}(\mathrm{x}, \mathrm{y})$ is also real and symmetric.
(b) Can you use the Fourier transform to compute the magnitude of the gradient for use in image differentiation? Justify your answer.
(c) A continuous Gaussian low pass filter in the continuous frequency domain has the transfer function

$$
\mathrm{H}(\mathrm{u}, \mathrm{v})=\mathrm{A} \exp \left[-\left(\mathrm{u}^{2}+\mathrm{v}^{2}\right) / 2 \sigma^{2}\right] .
$$

Show that the corresponding filter in the spatial domain is

$$
\mathrm{h}(\mathrm{x}, \mathrm{y})=\mathrm{A} \cdot 2 \pi \sigma^{2} \exp \left[-2 \pi^{2} \sigma^{2}\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)\right] .
$$

3. (a) The arithmetic decoding process is the reverse of the encoding procedure. Decode the message 0.23355 given the coding model.

| Symbol | Probability |
| :---: | :---: |
| a | 0.2 |
| e | 0.3 |
| i | 0.1 |
| o | 0.2 |
| u | 0.1 |
| $!$ | 0.1 |

(b) Define measure of similarity between two strings $a$ and $b$. Find its value, when
(i) all the symbols used in a and b are exactly the same,
(ii) all the symbols used in a and b are different.
4. (a) Use the LZW coding algorithm to encode the 7-bit ASCII string "aaaaaaaaaaa".
(b) Suppose that an image has the gray-level probability density functions shown below :


Here, $p_{1}(z)$ corresponds to objects and $p_{2}(z)$ corresponds to background. Assume that $\mathrm{p}_{1}=\mathrm{p}_{2}$ and find the optimal threshold between object and background pixels.
5. (a) Define the following :
(i) Image acquisition
(ii) Image compression
(iii) Morphological processing
(b) Find the normalized starting point of the code 41076765541322 .
(c) Find the expression for the signature of a rectangular boundary for the following figure :

6. (a) Give two boundary shapes that have the same mean and same third statistical moment descriptors, but different second moments.
(b) Consider a checkerboard image composed of alternating black and white squares, each of size $m \times m$. Give a position operator that would yield a diagonal co-occurrence matrix.
(c) Obtain the gray-level co-occurrence matrix of a $5 \times 5$ image composed of a checkerboard of alternating 1 's and 0 's, if the position operator $P$ is defined as
(i) "one pixel to the right" and
(ii) "two pixels to the right".

Assume that top left pixel has value 0 .
7. (a) The following pattern classes have Gaussian probability density functions:

$$
\begin{aligned}
& \mathrm{w}_{1}=\left\{(0,0)^{\mathrm{t}},(2,0)^{\mathrm{t}},(2,2)^{\mathrm{t}},(0,2)^{\mathrm{t}}\right\} \text { and } \\
& \mathrm{w}_{2}=\left\{(4,4)^{\mathrm{t}},(6,4)^{\mathrm{t}},(6,6)^{\mathrm{t}},(4,6)^{\mathrm{t}}\right\} .
\end{aligned}
$$

Assume that $\mathrm{P}\left(\mathrm{w}_{1}\right)=\mathrm{P}\left(\mathrm{w}_{2}\right)=\frac{1}{2}$ and obtain the equation of the Bayes' decision boundary between these two classes. Also, sketch the boundary.
(b) Describe the following :
(i) Fourier Descriptors
(ii) Homomorphic Filtering

