MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00998

Term-End Examination

June, 2015

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is **compulsory**. Answer any **four** out of the remaining six (Q. 2 to 7). Calculating devices are not allowed.*

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1. State whether the following statements are *true* or *false*, with a brief justification : *5×2=10*

(a) There is a unique graph of order 5 which is isomorphic to its complement.

(b) The Petersen graph is 2-colourable.

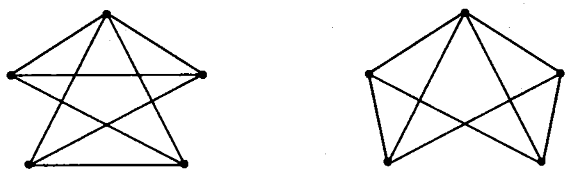
(c) The number of vertices of even degree in a graph must be odd.

(d) A graph of order n has at most $(n - 2)$ cut vertices.

(e) If a graph is 4-colourable, then it is planar.

2. (a) Prove that a connected graph is Eulerian, if and only if all its vertices have even degree. *6*

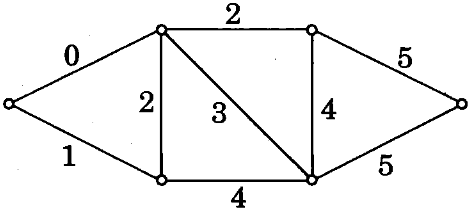
(b) Prove that the following two graphs are isomorphic : *4*



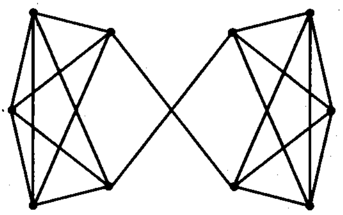
3. (a) If T is a tree with k edges and G is a simple graph with minimum degree at least k, then prove that T is a subgraph of G. *4*

(b) Compute the diameter and radius of the complete bipartite graph $K_{m,n}$, where $m \geq 2, n \geq 2$. *2*

(c) Apply Prim's algorithm to the following graph : *4*



4. (a) Define perfect matching in a graph and find a perfect matching in the following graph :



Also find $\alpha(G)$, $\beta(G)$, $\alpha'(G)$ and $\beta'(G)$ for the graph given above. *6*

(b) Prove that every component of the symmetric difference of two matchings is a path or an even cycle. *4*

5. (a) Define a hypercube Q_k and prove that Q_k is k-connected. *4*

(b) Test if the following sequence is graphic using Havel-Hakimi algorithm : *3*

5, 5, 5, 5, 5, 4, 4, 4, 4, 3, 3, 3, 2, 2, 1, 1

(c) If f is a feasible flow in a network and [S, T] is a source/sink cut, then prove that the net flow out of S and net flow into T equals val(f). *3*

6. (a) If G is an interval graph, show that $\chi(G) = \omega(G)$. *4*

(b) Draw a simple graph G with $\kappa(G) = 1$, $\kappa'(G) = 2$ and $\delta(G) = 3$. *2*

(c) Let G be a simple graph with at least three vertices and $\delta(G) \geq \frac{n(G)}{2}$. Prove that G is a Hamiltonian graph. *4*

7. (a) Prove that K_5 cannot be drawn on the plane without crossing of edges. *3*

(b) Prove that a graph G is bipartite, if and only if the dual G* is Eulerian. *4*

(c) Prove that a simple graph in G is 2-connected, if and only if for every triple (x, y, z) of distinct vertices, G has an x-z path through y. *3*
