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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

00518

M.Sc. (MACS)

Term-End Examination

June, 2015

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

***Note :** Answer any **five** questions. Use of calculator is **not** allowed.*

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1. (a) Explain the following terms giving an example of each : 2

(i) Static and Dynamic Models.

(ii) Deterministic and Stochastic Models.

(b) The return distribution on the two securities X and Y are given in the table below : 3

Possible Rates of Return		Associated Probability
X	Y	$p_{xj} = p_{yj}$
0·10	0·09	0·20
0·11	0·11	0·22
0·17	0·16	0·25
0·19	0·18	0·33

Find σ_{XY} and ρ_{XY}.

2. (a) In a population of animals, the proportionate birth rate and death rate are both constant, being 0·20 per year and 0·50 per year respectively. Formulate a model of population and discuss its long-term behaviour. *2*

(b) Following is the data for number of years students studied a subject and the score he/she received in that subject :

No. of years	3	4	4	2	5	3	4	5	3	2
Test score	57	78	72	58	89	63	73	84	75	48

Fit the least square line to this data. What is the score of the student who has studied for two years, according to this line ? *3*

3. Consider the following prey and predator interacting system under the effect of toxicant, where the concentration of the toxicant in the environment is assumed to be constant :

$$\frac{dN_1}{dt} = r_0 N_1 - r_1 C_0 N_1 - bN_1 N_2$$

$$\frac{dN_2}{dt} = - d_0 N_2 + \beta(C_0)\, bN_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

along with the initial conditions : $N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $C_0(0) = 0$ and $\beta(C_0) = \beta_0 - \beta_1 C_0$, where $\beta(C_0)$ is the conversion coefficient depending upon C_0. The variables and parameters notations in the above system of equations are :

$N_1(t)$ = Density of prey population.

$N_2(t)$ = Density of predator population.

$C_0(t)$ = Concentration of toxicant in the individuals of the population.

P = Concentration of toxicant in the environment and is constant.

Do the stability analysis of the system. *5*

4. Solve the following Integer programming problem using the branch and bound method : *5*

$$\text{Maximise } z = 3x_1 + 5x_2$$

$$\text{subject to } 2x_1 + 4x_2 \leq 25,$$

$$x_1 \leq 8,$$

$$2x_2 \leq 10,$$

and $x_1, x_2 \geq 0$ and are integers.

5. Consider the discrete time population model given by $N_{t+1} = \dfrac{rN_t}{1 + \left(\dfrac{N_t}{k}\right)^b}$, for a population N_t, where k is the carrying capacity of the population, r is the intrinsic growth rate and b is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $0 < r < 1$. Also find the first bifurcation value of the parmeter r. *5*

6. Explain each of the following with examples : *5*

(a) Reaction-diffusion model versus Advection – reaction – diffusion model.

(b) Variational matrix or Jacobian of a system of n differential equations.

(c) Data visualization.

(d) Hurwitz criteria.

(e) Multiple linear regression model with k predictors.