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MMT-008

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00558

June, 2015

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 hours *Maximum Marks : 100*

(Weightage : 50%)

Note : *Question no. 8 is* ***compulsory****. Answer any* ***six*** *questions from questions no.* ***1*** *to* ***7****. Use of calculator is* ***not*** *allowed.*

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1. (a) Consider M/M/1 queueing system with arrival rate λ and service rate 2λ and M/M/2 queueing system with arrival rate λ and service rate λ. Show that the average waiting time in system M/M/1 is smaller than the waiting time in M/M/2 system. 7

(b) Obtain the renewal equation when the inter arrival distribution is uniform on [0, 1]. Solve the equation for $t < 1$ also. 4

(c) Verify Chapman-Kolmogorov equation from the following transition matrix of a Markov Chain for m = n = 1 : *4*

$$P = \begin{bmatrix} 0 \cdot 5 & 0 \cdot 5 & 0 \\ 0 & 0 \cdot 5 & 0 \cdot 5 \\ 0 \cdot 5 & 0 & 0 \cdot 5 \end{bmatrix}.$$

2. (a) Determine the closed sets and mean recurrence times of the states of the Markov Chain with the following transition matrix : *7*

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{bmatrix} \end{array}.$$

(b) Let $\mathbf{X} \sim N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ where

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \text{ and } \boldsymbol{\Sigma} = \begin{pmatrix} 9 & 0 & 3 \\ 0 & 4 & 2 \\ 3 & 2 & 9 \end{pmatrix}.$$

(i) Find the marginal distribution of $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$.

(ii) Conditional distribution of X_1, given $X_2 = 1$ and $X_3 = 2$.

(iii) Coefficient of correlation between X_1 and X_3. 8

3. (a) Write the postulates of the birth and death process. Obtain the infinitesimal generator for birth and death process with birth rates λ_k and death rates μ_k. 7

(b) Two populations π_1 and π_2 have identical variances. A sample of size 40 was drawn from π_1 and a sample of size 50 was drawn from π_2. The summary statistics were

$$\bar{x}_1 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}, \qquad \bar{x}_2 = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}.$$

Test equality of means of both populations at 5% level of significance.

(You may use the values $F(2, 87)_{\cdot 05} = 3{\cdot}10$, $F(3, 87)_{\cdot 05} = 2{\cdot}71$) 8

4. (a) One of the two teller machines handles withdrawals only while the other handles deposits only in a bank. The service time of both the machines follows exponential distribution with mean service time 3 minutes. The depositors arrive in the bank at the rate of 16 per hour and withdrawers arrive at the rate of 14 per hour in Poisson fashion. Find the average waiting times of depositors and withdrawers in the queue. If each machine can handle both the jobs of deposits and withdrawals, then what will be the average waiting time in queue for a customer ? *9*

(b) Let $X = (X_1\ X_2\ X_3)' \sim N_3(\mu, \Sigma)$, where

$$\mu = (-1 \quad 1 \quad -1)' \text{ and } \sum = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix}.$$

$$\text{Let } Y = \begin{pmatrix} X_1 + X_2 + X_3 \\ X_1 + 2X_2 \end{pmatrix}.$$

Find l_2, such that $l' Y \sim N(0, 1)$. *6*

5. (a) Consider the mean vectors be $\mu_x = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\mu_y = 4$, and the covariance matrices of x_1, x_2 and y are

$$\sum\nolimits_{xx} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \ \sigma_{yy} = 9, \ \sigma_{xy} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

(i) Fit the equation $y = b_0 + b_1x_1 + b_2x_2$ as best linear fit equation.

(ii) Find the multiple correlation coefficient.

(iii) Find the mean square error. *8*

(b) Consider a branching process with offspring distribution given by

$$p_j = \begin{cases} 2/5, & j = 0 \\ 3/5, & j = 2. \end{cases}$$

Find the probability of extinction. *7*

6. (a) Consider two random variables X and Y whose joint probability mass function is given in the following table :

X \ Y	4	3	2
1	0·15	0·15	0·2
2	0·05	0·05	0·1
4	0·1	0·1	0·1

(i) Find E(X), E(Y), V(X) and V(Y).

(ii) Are X and Y independent ? Give reasons.

(iii) Find E(X/Y = 4) and V(X/Y = 4).

(iv) Find cov (X, Y). *10*

(b) A particular component in a machine is replaced instantaneously on failure. The successive component lifetimes are uniformly distributed over the interval [1, 4] years. Further, planned replacements take place every 3 years. Compute

(i) long-term rate of replacements,

(ii) long-term rate of failures,

(iii) long-term rate of planned replacements. *5*

7. (a) The variance-covariance matrix of three random variables X_1, X_2, X_3 is given by

$$\sum = \begin{bmatrix} 1 & 0.63 & 0.4 \\ 0.63 & 1 & 0.35 \\ 0.4 & 0.35 & 1 \end{bmatrix}.$$

Write its factor model. *9*

(b) Consider a 3-state Markov Chain with the transition matrix.

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

Find the stationary distribution of it. *6*

8. State whether the following statements are *true* or *false*. Justify your answer. *10*

(a) The matrix $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ is a variance-covariance matrix of two dimensional random variable.

(b) The quadratic form $x_1^2 - x_2^2$ is positive definite.

(c) The time between successive arrivals follows exponential distribution each with mean time 10 minutes, then number of arrivals follows Poisson distribution with mean 10 per hour.

(d) A subset C of the state space of a Markov Chain is closed, then any state in C can communicate with a state outside C.

(e) Principal components depend on the scales used to measure the variables.