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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00758

Term-End Examination

June, 2015

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is **compulsory**. Attempt any **four** questions out of questions no. **2** to **7**. Use of calculators is **not** allowed. All computations may be kept to three decimal places.*

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1. State whether the following statements are *true* or *false*. Justify your answer with the help of a short proof or a counter example. *5×2=10*

 (a) The explicit scheme :

 $$u_i^{n+1} = u_i^n + \lambda\,[u_{i+1}^n - 2u_i^n + u_{i-1}^n],$$

 $\lambda = k/h^2$, for solving the parabolic equation

 $u_t = u_{xx}$ is stable for $\lambda < 1$.

 (b) If F^{-1} denotes Fourier Inverse Transform, then

 $$F^{-1}\left[e^{-|\alpha|}\cos\alpha\right] = \frac{1}{2\pi}\left[\frac{1}{(x+1)^2+1} + \frac{1}{(x-1)^2+1}\right].$$

(c) The function y(t) satisfying the integral equation

$$y(t) + \int_0^t y(z) \, . \, (t - z) \, dz = t$$

is y(t) = sin t.

(d) If $J_n(x)$ is Bessel's function, then

$$J_1'(x) = - J_0(x).$$

(e) For $h > 0.0278$ the fourth order Runge-Kutta method when used to solve the initial value problem

$$y' = - 100 \, y, \; y(0) = 1$$

produces stable result.

2. (a) Find the series solution, near x = 0, of the differential equation

$$x(1 - x) \, y'' + (1 - x) \, y' - y = 0.$$ 7

(b) Using Runge-Kutta fourth order formula, with $h = 0{\cdot}2$, solve $\frac{dy}{dx} = 1 + y^2$ for $y(0{\cdot}2)$. 3

3. (a) Show that $\int_0^{\infty} e^{-st} \, L_n(t) \, dt = \frac{1}{s}\left(1 - \frac{1}{s}\right)^n$, where $L_n(t)$ is a Laguerre polynomial. 3

(b) Using Fourier transforms, solve the differential equation

$$y' - 5y = H(x) \, e^{-2x}, \; - \infty < x < \infty$$

where H(x) is the unit step function. 3

(c) Using Schmidt method, with $\lambda = \frac{1}{6}$, find the solution of initial value problem $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ with u(0, t) = 0 = u(1, t) and

$$u(x, 0) = \begin{cases} 2x & \text{for} \quad 0 < x < 1/2 \\ 2(1-x) & \text{for} \quad 1/2 < x < 1, \end{cases}$$

with h = 0·2. *4*

4. (a) Prove that

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{1}{x} \sin x - \cos x \right].$$ *3*

(b) Find the Laplace transform of the function

$$f(t) = \begin{cases} 3t, & 0 < t < \pi \\ \pi - 2t, & \pi < t < 2\pi. \end{cases}$$ *3*

(c) Solve the boundary value problem $\frac{d^2 y}{dx^2} = y$ with y(1) = 1 and y(0) = 0, using second order finite difference method with $h = \frac{1}{3}$. *4*

5. (a) Using Laplace Transforms, solve $\frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}$ given that u(0, t) = u(2, t) = 0, $u_t(x, 0) = 0$ and $u(x, 0) = 10 \sin 2\pi x$. *5*

(b) Find the solution of boundary value problem $\nabla^2 u = 0$ in R, where u = x + y on the boundary, using Galerkin method with triangular element. Take $h = \frac{1}{2}$ and R is the square $0 \leq x \leq 1, \ 0 \leq y \leq 1$. 5

6. (a) Construct the Green's function for the boundary value problem

$$y'' - 16y = 0, \ y(0) = 0 = y(1).$$ 5

(b) The five point formula for Poisson equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = G(x, y)$ is

$$u_{i+1, j} + u_{i-1, j} + u_{i, j+1} + u_{i, j-1} - 4u_{i, j} = h^2 G(x_i, y_j)$$

Using Taylor series expansions, find the order of this five-point formula. 5

7. (a) Find Fourier Cosine transform of the function 3

$$f(t) = \begin{cases} t & , \quad 0 < t < 1/2 \\ 1 - t & , \quad 1/2 < t < 1 \\ 0 & , \quad t > 1. \end{cases}$$

(b) Heat conduction equation $u_t = u_{xx}$ is approximated by the method

$$u_m^{n+1} - u_m^{n-1} = \frac{2k}{h^2} \delta_k^2 u_m^n.$$

Investigate the stability of this method using Von Neumann Method. *4*

(c) Solve the initial value problem $y' = -2xy^2$, $y(0) = 1$ with $h = 0{\cdot}2$ on the interval $[0, 0{\cdot}4]$ using the predictor-corrector method

$$P : y_{k+1} = y_k + \frac{h}{2}(3y'_k - y'_{k-1})$$

$$C : y_{k+1} = y_k + \frac{h}{2}(y'_{k+1} + y'_k).$$

Perform two corrector iterations per step. Use the exact solution $y(x) = \dfrac{1}{1+x^2}$ to obtain the starting value. *3*