No. of Printed Pages: 3

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2015

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MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours

Maximum Marks: 25

- Note: Question no. 1 is compulsory. Attempt any three questions from questions no. 2 to 5. Use of calculator is **not** allowed.
- 1. State giving reasons whether the following statements are *true* or *false*: $5 \times 2=10$
 - (a) The function f(z)

$$=\frac{(x^{3}-y^{3})+i(x^{3}+y^{3})}{x^{2}+y^{2}}, \ z\neq 0,$$

f(0) = 0, is differentiable at 0.

(b) Product of two harmonic functions is harmonic.

MMT-005

1

P.T.O.

(c) The function $f(z) = \frac{\sin z}{z^3}$, $z \neq 0$, has a pole of order 3 at z = 0.

(d) If C:
$$|z| = 1$$
 and $f(z) = \frac{\sin z}{\left(z - \frac{\pi}{6}\right)(z - 2)}$ then

$$\oint_C f(z) dz = 2\pi i.$$

(e)
$$\sin\left(\frac{1}{z}\right)$$
 has a simple pole at $z = 0$.

2. (a) State Cauchy Riemann equations in Polar form and use them to find the harmonic conjugate of $u(r, \theta) = ln r$, r > 0.

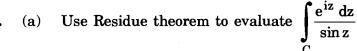
$$f(z) = \frac{1}{z^2 - 3z + 2}$$
 in the region $1 < |z| < 2$. 2

3

3

3. Evaluate
$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^3}$$
 using contour integration. 5

4.



where C is a positively oriented quadrilateral with vertices $\pm 2 \pm 3i$.

MMT-005

2

 $\sum_{n=0}^{\infty} \frac{n! z^n}{n^2}$. Also find the domain of

convergence of the series.

- **5.** (a)
- If f(z) is an entire function such that

 $|f(z)| \le e^{\text{Im } z} \forall z$, show that $f(z) = ae^{-iz}$ where $|a| \le 1$.

(b) Prove that Möbius transformation

$$w = \frac{2z-1}{2-z}$$
 maps unit disc to itself.

3

2

2

MMT-005