No. of Printed Pages : 3

MMT-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

June, 2015

00898

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from questions no. **2** to **5**. Use of calculator is **not** allowed.*

1. State giving reasons whether the following statements are *true* or *false* : *5× 2=10*

   (a) The function f(z)

   $$= \frac{(x^3 - y^3) + i(x^3 + y^3)}{x^2 + y^2}, \ z \neq 0,$$

   f(0) = 0, is differentiable at 0.

   (b) Product of two harmonic functions is harmonic.

(c) The function $f(z) = \frac{\sin z}{z^3}$, $z \neq 0$, has a pole of order 3 at $z = 0$.

(d) If $C : |z| = 1$ and $f(z) = \frac{\sin z}{\left(z - \frac{\pi}{6}\right)(z-2)}$ then

$$\oint_C f(z)\, dz = 2\pi i.$$

(e) $\sin\left(\frac{1}{z}\right)$ has a simple pole at $z = 0$.

2. (a) State Cauchy Riemann equations in Polar form and use them to find the harmonic conjugate of $u(r, \theta) = \ln r$, $r > 0$. *3*

(b) Find Laurent's expansion of

$f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$. *2*

3. Evaluate $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^3}$ using contour integration. *5*

4. (a) Use Residue theorem to evaluate $\int_C \frac{e^{iz}\, dz}{\sin z}$

where C is a positively oriented quadrilateral with vertices $\pm 2 \pm 3i$. *3*

(b) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n!\, z^n}{n^2}.$$ Also find the domain of convergence of the series. 2

5. (a) If f(z) is an entire function such that

$|f(z)| \leq e^{\text{Im}\, z}\ \forall\ z$, show that $f(z) = ae^{-iz}$ where $|a| \leq 1$. 2

(b) Prove that Möbius transformation

$w = \dfrac{2z-1}{2-z}$ maps unit disc to itself. 3