No. of Printed Pages : 3

MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00038

June, 2015

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* **5** *is* ***compulsory****. Answer any* ***three*** *questions from questions no. 1 to 4. Use of calculator is* ***not*** *allowed.*

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1. (a) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be given by

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y-z \\ x-y+z \end{bmatrix}.$$ Find the matrix of T

with respect to the bases

$$\left\{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right\} \text{ and } \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}.$$

Is T invertible ? Justify your answer. *3*

(b) The following matrix equation describes the migration pattern from City A to City B :

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 0{\cdot}95 & 0{\cdot}15 \\ 0{\cdot}05 & 0{\cdot}85 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}.$$

Here x_n, y_n denote the populations in City A and City B, respectively, after n years. What will the long term effect of the migration be on the population of the cities ? *2*

2. (a) Write the Jordan canonical form for the matrix *2*

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

(b) Find a least square solution for the system : $y + z = 1$, $-x + y + 2z = 0$, $2y + 2z = 1$, $x + y = 0$. *3*

3. (a) Solve the system of differential equations

$$\frac{dy(t)}{dt} = Ay(t) \quad \text{with} \quad y(t) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ *3*

(b) Find the square root of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. *2*

4. Write the singular value decomposition for the matrix $\begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix}$. *5*

5. Which of the following statements are *true* and which are *false* ? Give reasons for your answers.

5×2=10

(a) The sum of two diagonalizable matrices is also a diagonalizable matrix.

(b) There is a matrix with characteristic polynomial $(x^2 - 1)^2$ and the minimal polynomial $(x + 1)^2$.

(c) There is no unitary matrix with a column $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(d) The geometric multiplicity of eigenvalue 1 for $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is 2.

(e) If A is $m \times n$ matrix, then $A^* A$ is positive semi-definite.