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BICE-027

B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

00396

Term-End Examination

June, 2015

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Answer any **ten** questions. All questions carry equal marks. Use of scientific calculator is permitted.*

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1. Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \quad -\pi < x < \pi.$$

Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}.$$ 7

2. If $f(x) = x + x^2$ for $-\pi < x < \pi$, and $f(x) = \pi^2$ for $x = \pm\pi$. Expand f(x) in Fourier series. 7

3. Express $f(x) = x$ as a half-range sine series in $0 < x < 2$. 7

4. If $f(x) = x$, for $0 < x < \frac{\pi}{2}$

$$= \pi - x, \text{ for } \frac{\pi}{2} < x < \pi,$$

show that

$$f(x) = \frac{4}{\pi}\left[\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \ldots\right].$$ 7

5. Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0 & |x| > 1. \end{cases}$$

Hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cdot \cos\frac{x}{2}\, dx.$$ 7

6. Obtain Fourier sine transform of

$$f(x) = \begin{cases} x & \text{for} \quad 0<x<1 \\ 2-x & \text{for} \quad 1<x<2 \\ 0 & \text{for} \quad x>2. \end{cases}$$ 7

7. Solve : 7

$$x(y^2 - z^2)\, p + y\,(z^2 - x^2)\, q - z\,(x^2 - y^2) = 0$$

8. Solve : 7

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$$

9. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t. 7

10. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form

$$y = a \sin\left(\frac{\pi x}{l}\right)$$

from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi c t}{l}\right).$$ 7

11. The temperature at one end of a bar, 50 cm long with insulated sides, is kept at 0°C and that the other end is kept at 100°C until steady-state conditions prevail. The two ends are then suddenly insulated, so that the temperature gradient is zero at each end thereafter. Find the temperature distribution. 7

12. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by

$$y = y_0 \sin^3 \left(\frac{\pi x}{l} \right).$$

If it is released from rest from this position, find the displacement $y(x, t)$. 7

13. A square plate is bounded by the lines

$$x = 0, y = 0, x = 20 \text{ and } y = 20.$$

Its faces are insulated. The temperature along the upper horizontal edge is given by

$$u(x, 20) = x(20 - x), \text{ when } 0 < x < 20,$$

while the other three edges are kept at 0°C. Find the steady state temperature in the plate. 7

14. The bounding diameter of a semi-circular plate of radius a cm is kept at 0°C and the temperature along the semi-circular boundary is given by

$$u(a, \theta) = \begin{cases} 50\,\theta & \text{when } 0 < \theta \nless \frac{\pi}{2} \\ 50(\pi - \theta) & \text{when } \frac{\pi}{2} < \theta < \pi. \end{cases}$$

Find the steady-state temperature function $u(r, \theta)$. 7

15. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

subject to the conditions

$u(0, y) = u(l, y) = u(x, 0) = 0$

and $u(x, a) = \sin \frac{n\pi}{l} x$. 7