No. of Printed Pages : 7

ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

01487

June, 2015

ET-101 (A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of calculator is allowed.*

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1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate (any ***one***).

(i) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$

(ii) $\lim\limits_{x \to 0} x^{2x}$

(b) Show that the tangent at (a, b) to the curve

$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 2 \text{ is } \frac{x}{a} + \frac{y}{b} = 2.$$

(c) Find $\frac{dy}{dx}$, where $y = \tan^{-1} \frac{\sqrt{1+x^2} - 1}{x}$.

(d) A gardener having 120 m of fencing wishes to enclose a rectangular plot of land and also to erect a fence across the land on all the four sides. What is the maximum area that can be enclosed ?

(e) If $y = \sin(m \sin^{-1} x)$, show that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0.$$

(f) If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$, show that

$$x \frac{\partial u}{\partial y} + y \frac{\partial u}{\partial y} = 0.$$

(g) Examine the continuity of the function g(x) at x = 0,

$$\text{where } g(x) = \begin{cases} \tan^{-1} x, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(h) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, compute $\frac{dy}{dx}$.

2. Answer any ***four*** of the following : *4×4=16*

(a) By using Simpson's $\frac{1}{3}$rd rule, evaluate

$$\int_0^1 \frac{1}{1+x^2}\, dx.$$

(b) Evaluate (any ***one***).

(i) $\int e^x \frac{1 + \sin x}{1 + \cos x}\, dx$

(ii) $\int \sin^7 x\, dx$

(c) Show that if $x > 0$, then $x + \frac{1}{x} \geq 2$.

(d) Prove that $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\, dx = \frac{\pi}{4}$.

(e) Find the area between the parabola $y^2 = 8x$ and the straight line $x = 2$.

(f) Solve (any ***one***).

(i) $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

(ii) $(x^2 + y^2)\, dy = xy\, dx$

3. Answer any ***four*** of the following : *4×4=16*

(a) Find a unit vector parallel to the resultant of vectors $\mathbf{r}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, $\mathbf{r}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.

(b) Determine the value of a so that

$\mathbf{A} = 2\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}$, and

$\mathbf{B} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ are perpendicular.

(c) Find the projection of the vector $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ on the vector

$\mathbf{B} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$.

(d) Find the volume of the parallelopiped whose edges are represented by

$\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, $\mathbf{B} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, and

$\mathbf{C} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.

(e) If $\mathbf{A} = xz^3\hat{\mathbf{i}} - 2x^2yz\hat{\mathbf{j}} + 2yz^4\hat{\mathbf{k}}$, find $\nabla \times \mathbf{A}$ (or curl **A**) at the point (1, –1, 1).

(f) Show that $\mathbf{F} = (2xy + z^3)\hat{\mathbf{i}} + x^2\hat{\mathbf{j}} + 3xz^2\hat{\mathbf{k}}$, is a conservative force field. Find the scalar potential. Also compute the work done in moving an object in this field from (1, –2, 1) to (3, 1, 4).

4. Answer any ***six*** of the following : *6×3=18*

(a) Given $A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & -4 \end{bmatrix}$ and

$$B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \\ 5 & 2 & -1 \end{bmatrix}.$$

Find where possible

A + B, A − B, AB and BA, stating the reasons where operations are not possible.

(b) Find $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$; from the matrix equation

AX = B, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}.$$

(c) Prove without expanding the determinant

that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$.

(d) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^2 and show

that $A^2 = A^{-1}$.

(e) Solve by Cramer's rule :

$2x - y + z = 3$

$4x + 2y + 3z = 7$

$6x + 7y + 5z = 11$

(f) Find the inverse of the matrix

$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$.

(g) Show that

$$\begin{vmatrix} 2ab & a^2 & b^2 \\ b^2 & 2ab & a^2 \\ a^2 & b^2 & 2ab \end{vmatrix}$$ is a perfect square.

(h) Solve for x, y, z and t where

$$\begin{bmatrix} y+z & z+x \\ 7-t & 6-z \end{bmatrix} = \begin{bmatrix} 9-t & 8-t \\ x+y & x+y \end{bmatrix}.$$